GROSS PANEL ANALYSIS OF SHIP STIFFENER-PLATING STRUCTURES

NELSON G. VASQUEZ GERMAIN
AND
ALLAN K. CAMERON, JR.

GROSS PANEL ANALYSIS OF

SHIP STIFFENER-PLATING STRUCTURES

by

NELSON G. VASQUEZ GERMAIN, LIEUTENANT, VENEZUELAN NAVY
B.C.E., Rensselaer Polytechnic Institute
(1955)

and

ALLAN K. CAMERON, JR., LIEUTENANT, UNITED STATES NAVY
B.S., United States Naval Academy
(1950)

SUBMITTED IN PARTIAL FULFILLMENT

OF THE REQUIREMENTS FOR THE

DEGREES OF NAVAL ENGINEER

AND MASTER OF SCIENCE

at the

MASSACHUSETTS INSTITUTE OF

TECHNOLOGY

June, 1958

ABSTRACT

GROSS PANEL ANALYSIS OF SHIP STIFFENER-PLATING STRUCTURES.

by NELSON G. VASQUEZ GERMAIN and ALLAN K. CAMERON, JR.

Submitted to the Department of Naval Architecture and Marine Engineering on May 26, 1958 in partial fulfillment of the requirements for the degrees of Naval Engineer and Master of Science in Naval Architecture and Marine Engineering.

The design of ship hull structure is presently based on extrapolation from existing successful prototypes. Adequacy is checked by laborious analytical calculations. Other branches of the science of structural design have been able to develop methods of design by snythesis as opposed to uneconomical cut and try analysis.

The object of our study was to develop additional means of ship structural design by synthesis. It was intended that given gross panel proportions and condition of loading the type, size, orientation and spacing of stiffeners and thickness of plating could be determined. Criteria for selection were based on minimum weight of the gross panel.

The case studied in detail was that of rectangular gross panels, with uniform edge compression in the plane of the panel perpendicular to the stiffener orientation. Tee stiffeners were used in the calculations. The results were obtained analytically and are presented in the curves contained herein. A method is recommended by which a rapid iterative solution for minimum weight and adequate strength is obtained in dimensional form.

Thesis Supervisor:	J. Harvey Evans
Title:	Associate Professor of Naval Architecture and Marine Engineering

ACKNOWLEDGMENT

This work is intended to supplement that of L.A. Harlander in his thesis "Optimum Plate Stiffener Arrangement for Various Types of Loading". The suggestion that ship structural design be approached as a series of individual gross panels was made by Professor J. Harvey Evans of the Department of Naval Architecture and Marine Engineering at the Massachusetts Institute of Technology. His encouragement and valuable suggestions are deeply appreciated.

TABLE OF CONTENTS

INTRODUCTION

Weight minimization in structural design is by no means an unusual problem. It is of extreme importance in aircraft design. In ship design a lighter structure means a higher payload or cargo deadweight for a given displacement, or a reduced displacement and power for given payload.

If the cost of erection were constant for all structures, or based only on the structural weight, the lightest structure would also be the least expensive. Such is not the case, and certain structures might be prohibitively expensive to fabricate due to labor and machine requirements. It is generally true that warship design accepts somewhat higher fabrication and erection expense in order to economize weight as compared with commercial vessels. Weight minimization at least indicates a tendency toward overall economy, and gives an advanced basis for further comparison or compromise to reach an overall optimum.

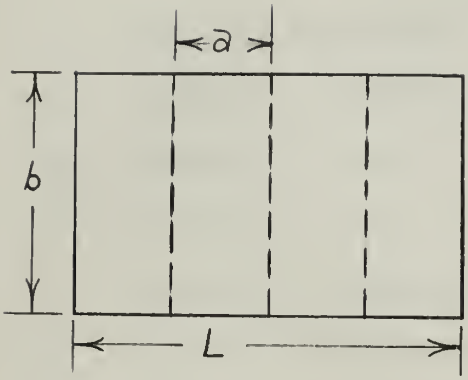
Ship structures are largely made up of numbers of panels, generally approaching rectangular shape. Such panels are bounded by bulkheads, decks, side or bottom shell and deep girders such as web frames. For example, such a gross panel might be the deck of a commercial vessel between the two sides and two adjacent transverse bulkheads. In this case

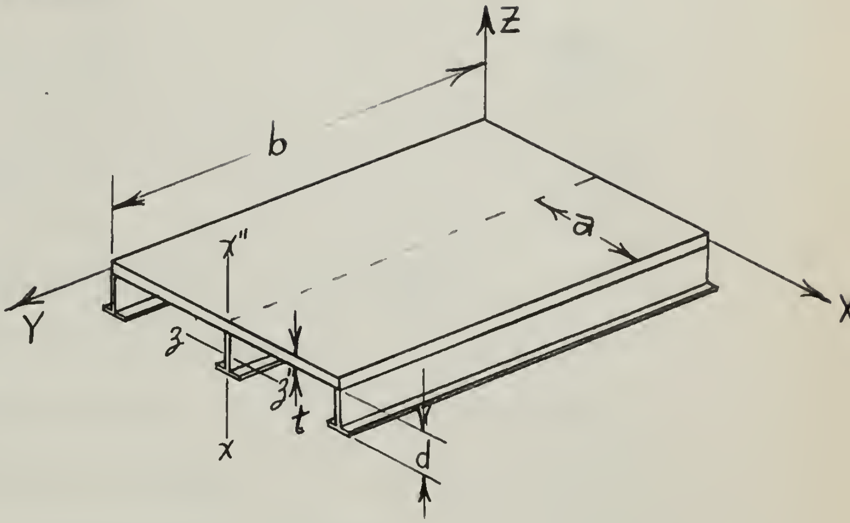
the stiffeners would be the transverse deck beams. One of the bounding bulkheads could have a gross panel bounded by two adjacent decks and by the side shells or longitudinal bulkheads if fitted. The stiffeners in this case would be the vertical bulkhead stiffeners.

Thus, study of gross panels could provide a basis for ship structural design. It is realized that such gross panels are subjected to numerous types of loading, often in combination. Bottom or side shell structure for example might be subjected to lateral hydrostatic pressure, edge compression and tension in the plane of the panel in one or two directions, and shear. Such loads may be uniformly distributed or vary in analytic or non-analytic manner. Hence it is not surprising that design has been and in great measure is typically a process of trial and error, and that the safety factors though comparatively large are not definitely known.

The objective of this work is to develop a method for design synthesis of gross panels on a basis of least weight. Since Harlander had already studied the cases of 1) uniform hydrostatic lateral loading and 2) edge compression parallel to the stiffeners, the next logical step is the study of edge compression perpendicular to the stiffeners.

SYMBOLS





		Unit
b	Plate dimension parallel to stiffener	inches
a	Stiffener spacing	inches
L	Plate dimension perpendicular to stiffener	inches
t	Plate thickness	inches
α	Aspect Ratio = a/b	--
ζ	Coefficient of restraint	--
ν	Poisson's ratio	--
$\bar{k}$	Plate factor	--
I_{zz}'	Moment of inertia of stiffener about y-y axis (I_{yy} in ref. 6, p. 39)	in^4
I_{xx}''	Moment of inertia of stiffener about x-x axis (I_{xx} in ref. 6, p. 39)	in^4
d	Depth of stiffener	in
K	Torsion constant of stiffener	in^4

SYMBOLS
(continued)

		Unit
A	Area of stiffener	in^2
Ab	Volume of stiffener	in^3
abt	Volume of plate	in^3
V	Volume of plate and stiffener in one panel ...	in^3
n	Number of subpanels in gross panel	--
σ_c	Critical stress in buckling	psi
E	Modulus of plasticity of steel	psi
E_t	Tangent modulus of elasticity of steel	psi
τ	$= \frac{E_t}{E}$	--
c	= thickness-width ratio = t/a	--
ω	= deflection in Z direction	in

PROCEDURE

Panel Analysis

The solution is based on the so-called Small Deflection Theory. That is, the deflection w is small compared with the thickness t of the plate. The fundamental equation is that due to St. Venant (1) and it is:

$$\frac{EI}{1-\nu^2}\left(\frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4}\right) + t\left(\sigma_x \frac{\partial^2 w}{\partial x^2} + \sigma_y \frac{\partial^2 w}{\partial y^2} + 2\tau_{xy}\frac{\partial^2 w}{\partial x \partial y}\right)=0 \qquad \text{(a)}$$

If a uniformly distributed compressive load along the b edges is considered, the stress σ_x becomes constant, and terms involving σ_y and τ_{xy} vanish. Then equation (a) takes the simplified form

$$\frac{EI}{1-\nu^2}\left(\frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4}\right) + \sigma_x t \frac{\partial^2 w}{\partial x^2} = 0 \qquad \text{(b)}$$

Equation (b) is valid only within the limit of Hooke's law and therefore has to be modified when σ_x exceeds the proportional limit of the material being considered. For values of σ_x above the proportional limit, the Tangent-Modulus Theory is used. The theory implies that when σ_x

exceeds the proportional limit, the tangent-modulus E_t will be effective in the x and y directions. (3),(4). A close, conservative (3) approximation is obtained and the solution of the differential equation is greatly simplified by this assumption. The differential equation is reduced to:

$$D\tau\left(\frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4}\right) + \sigma_x t \frac{\partial^2 w}{\partial x^2} = 0 \qquad (c)$$

Where $\tau = \frac{E_t}{E}$ and $D = \frac{EI}{1-\nu^2}$

The complete solution is given in Reference (3), Chapter XII, Article 116.

Our study was limited to the case of stiffened plating in edge compression. For large aspect ratios of subpanels (the length in the direction of compressive forces much larger than the width) it can be shown (3) that restraint at the loaded edges has little effect on the buckling strength of the panel. For small aspect ratios of subpanels (a/b less than unity) the effect of restraint at the loaded edges becomes significant. The loaded edges, therefore, cannot be considered as either clamped or simply supported. Then, the loaded edges were assumed to be elastically restrained. The elastic restraint is supplied by the torsional rigidity of the stiffeners at the loaded edges.

The unloaded edges are assumed to be simply supported. Some degree of restrain will always exist in an actual case. In any case, the assumption is conservative and simplifies greatly the solution. The smallest value of the critical buckling stress σ_c occurs with a deflection of the plate in one half wave in the direction parallel to the y-axis.

Solving the differential equation (c) and by proper use of the boundary conditions, we obtain the following formula:

$$\frac{\sigma_c}{\tau} = \frac{\pi^2 E}{12(1-\nu^2)} \left(\frac{t}{a}\right)\bar{k} \qquad \text{(d)}$$

The critical buckling stress σ_c is then found from Table III which corrects for the tangent modulus.

$\bar{k}$ is a plate factor, a function not only of the aspect ratio $\alpha = \frac{a}{b}$ but also of the stiffeners used, by means of the coefficient of restraint, ζ , where

$$\zeta = \frac{t^3 b^2}{53.9a\left(\frac{\pi^2 I d^2}{b^2} + \frac{K}{2.6}\right)} \qquad \text{(e)}$$

Theoretically ζ can assume values from zero to infinity. When $\zeta = 0$, the plate is completely fixed (clamped) at the loaded edges and when $\zeta = \infty$ the plate is free to rotate about these edges (simply supported).

Values of $\bar{k}$ as function of $\int$ and α are tabulated in Table I (3). These values were plotted on a large scale graph for accurate interpolation.

These expressions involve the characteristics of the plating and sections to be used in fabricating the panels. Concurrently with the effort to arrive at an analytic expression for panel weight in terms of scantlings, it was attempted to extend this to panel buckling through the foregoing expressions. The relations arrived at in the latter case were found to be unmanageable forcing resort to an iterative solution.

In order to pursue such solution, it was necessary to arrive at a series of values to cover the applicable range of the variables. The variables include:

a) Plating thickness.

b) Stiffener spacing (panel width a) through aspect ratios a/b.

c) Panel length b.

d) Stiffener rigidity through size and proportions.

<u>Thickness of Plating</u>

Time permitted selection of only the four different thicknesses of 1/4 inch, 3/8 inch, 1/2 inch, 3/4 inch. These values range from the least normally used in ship structures to about the middle of the range for large vessels.

Aspect Ratio Selection

The aspect ratios, α, of subpanel studied were: 0.1, 0.2, 0.3, 0.4, 0.5, and 0.8. This range extends beyond that normally appearing in ships.

Sub-Panel (stiffener) Length

A range of values from the minimum normally experienced to near the maximum was selected. These lengths were 5', 7½', 10', 15', 20'-10" and 30 feet. The values were increased by about fifty per cent in each case in order to adequately cover the desired range and permit reasonably accurate interpolation for intermediate values.

Stiffener Selection

One of the basic problems that an investigation of this type presents is the selection of stiffeners which are to be used. An ideal type of stiffener could have been selected in such a way as to facilitate the mathematical analysis. However, in order to have any practical application, available rolled sections should be used. The principal types of stiffeners in use are flat bars, angles and tees. The purpose was two-fold. First, tee stiffeners are widely used in Naval practice which is our principal field of interest; second, the tee stiffener is the most efficient of the three types mentioned since it has a better distribution of weight for a

higher moment of inertia. However, the method evolved by this study could be applied to any type of stiffener used in Naval and commercial practice.

Eight stiffeners were selected with the depth of tees varying from four inches to twelve inches, and the weight per foot from three and a quarter pounds to thirty-eight pounds.

From the list of tee sections recommended by the section modulus charts prepared by Buships, the lightest adequate sections of each successive depth were selected. These were supplemented by shapes from (6). The range does not include some of the heavier stiffeners found in current practice.

The characteristics of stiffeners are shown in Table II.

RESULTS

Implicit in our method of solution and its application are several assumptions concerning the data available to the designer and the information he desires. Initially it is assumed that he knows the gross panel dimensions and the loading (per running foot, in the plane of the plating and perpendicular to the stiffeners), and is willing to utilize the family of stiffeners specified. On a basis of arbitrarily selected number of sub-panels in the gross panel and plating thickness, the information available includes the critical buckling stress for comparison with the operating stress desired, and sub-panel weight. Thence the weights of adequate gross panel combinations can be arrived at for selection of the optimum arrangement.

For each length of stiffener (b), each aspect ratio of sub-panel (α), and each thickness (t), the effect of each of the family of stiffeners upon buckling stress was computed. Typical computations are shown in Table IV. The results are shown in Tables V through XVI, and Figures I through XXIV.

TABLE V

Results - Buckling Stress versus Sub-Panel Weight

σ_c = lb/in^2 x 10^{-3} V - in^3 x 10^{-3} b = 60"

a/b = .1 STIFFENER

Plate Thickness		A	B	C	D	E	F	G	H
1/4	σ_c	31.2	31.7	31.8	31.8	31.9	31.9	31.9	3.19
	V	1.5	.24	.28	.36	.41	.53	.64	.76
3/8	σ_c	31.9	32.2	32.3	32.4	32.4	32.5	32.5	32.5
	V	.19	.28	.33	.40	.45	.58	.68	.81
1/2	σ_c	32.2	32.4	32.5	32.6	32.7	32.7	32.8	32.8
	V	.24	.33	.37	.44	.50	.62	.73	.85
3/4	σ_c	32.6	32.7	32.7	32.8	32.8	32.9	32.9	33.0
	V	.33	.42	.46	.54	.59	.71	.82	.94

a/b = .2 STIFFENER

Plate Thickness		A	B	C	D	E	F	G	H
1/4	σ_c	27.3	28.3	28.5	28.6	28.7	28.7	28.8	28.8
	V	.24	.33	.37	.44	.50	.62	.73	.86
3/8	σ_c	29.3	30.5	30.8	30.9	31.0	31.0	31.1	31.1
	V	.33	.42	.46	.54	.59	.71	.82	.94
1/2	σ_c	30.2	31.1	31.4	31.6	31.7	31.8	31.8	31.9
	V	.42	.51	.55	.62	.68	.80	.91	1.03
3/4	σ_c	31.5	31.8	32.0	32.1	32.3	32.4	32.4	32.5
	V	.60	.69	.73	.80	.86	.98	1.09	1.21

a/b = .3 STIFFENER

Plate Thickness		A	B	C	D	E	F	G	H
1/4	σ_c	17.6	20.3	20.9	21.2	21.4	21.5	21.6	21.6
	V	.33	.42	.46	.54	.59	.71	.82	.94
3/8	σ_c	26.2	28.1	28.5	28.7	28.8	28.9	28.9	29.0
	V	.46	.56	.60	.67	.72	.85	.95	1.08
1/2	σ_c	27.9	29.5	30.0	30.3	30.4	30.6	30.6	30.6
	V	.60	.69	.73	.80	.86	.98	1.09	1.21
3/4	σ_c	30.0	30.8	31.1	31.4	31.6	31.8	31.8	31.9
	V	.87	.96	1.00	1.08	1.13	1.25	1.36	1.48

TABLE VI

Results - Buckling Stress versus Sub-Panel Weight

σ_c = lb/in^2 x 10^{-3} V - in^3 x 10^{-3} b = 60"

a/b = .4

Plate Thickness		STIFFENER A	B	C	D	E	F	G	H
1/4	σ_c	10.8	12	12.3	12.4	12.5	12.5	12.6	12.6
	V	.42	.51	.56	.62	.68	.80	.91	1.03
3/8	σ_c	18.8	24.5	25.4	25.6	25.8	25.9	25.9	26.0
	V	.60	.69	.74	.80	.86	.98	1.09	1.21
1/2	σ_c	25.3	27.4	28.2	28.5	28.7	28.8	28.9	29.0
	V	.78	.87	.92	.98	1.04	1.16	1.27	1.39
3/4	σ_c	28.3	29.5	30.2	30.5	30.8	30.9	31.1	31.1
	V	1.14	1.23	1.28	1.34	1.40	1.52	1.63	1.75

a/b = .5

Plate Thickness		STIFFENER A	B	C	D	E	F	G	H
1/4	σ_c	7.5	8.2	8.3	8.4	8.4	8.4	8.4	8.4
	V	.51	.60	.64	.72	.77	.89	1.00	1.12
3/8	σ_c	13.7	17.0	17.9	18.3	18.6	18.7	18.9	19.0
	V	.73	.82	.87	.94	.99	1.12	1.22	1.35
1/2	σ_c	19.2	25.5	26.2	26.6	26.8	26.9	27.0	27.0
	V	.96	1.05	1.09	1.16	1.22	1.34	1.45	1.57
3/4	σ_c	26.8	28.4	29.2	29.6	29.9	30.1	30.2	30.3
	V	1.42	1.50	1.54	1.62	1.67	1.79	1.90	2.02

a/b = .8

Plate Thickness		STIFFENER A	B	C	D	E	F	G	H
1/4	σ_c	3.8	4.0	4.0	4.0	4.0	4.0	4.0	4.1
	V	.78	.87	.91	.98	1.04	1.16	1.27	1.39
3/8	σ_c	7.7	8.6	8.9	9.0	9.0	9.1	9.1	9.1
	V	1.14	1.23	1.27	1.34	1.40	1.52	1.63	1.75
1/2	σ_c	10.9	14.3	15.2	15.6	15.8	16.0	16.1	16.3
	V	1.50	1.59	1.64	1.70	1.76	1.88	1.99	2.11
3/4	c	21.4	25.5	26.3	26.8	27.0	27.3	27.4	27.5
	V	2.22	2.31	2.36	2.42	2.48	2.60	2.71	2.83

TABLE VII

Results - Buckling Stress versus Sub-Panel Weight

σ_c = lb/in^2 x 10^{-3} V - in^3 x 10^{-3} b = 90"

a/b = .1 — STIFFENER

Plate Thickness		A	B	C	D	E	F	G	H
1/4	σ_c	28.3	29.8	30.2	30.3	30.4	30.5	30.5	30.6
	V	.29	.42	.49	.60	.68	.86	1.02	1.21
3/8	σ_c	29.9	30.9	31.4	31.6	31.7	31.8	31.8	31.9
	V	.39	.52	.60	.70	.78	.96	1.12	1.31
1/2	σ_c	31.0	31.4	31.8	32.0	32.1	32.2	32.3	32.3
	V	.49	.62	.70	.80	.88	1.07	1.22	1.41
3/4	σ_c	32.0	32.1	32.2	32.3	32.4	32.5	32.6	32.7
	V	.69	.82	.90	1.00	1.08	1.27	1.43	1.61

a/b = .2 — STIFFENER

Plate Thickness		A	B	C	D	E	F	G	H
1/4	σ_c	14.0	18.2	19.6	20.0	20.5	20.8	20.9	21.0
	V	.49	.62	.70	.80	.88	1.07	1.22	1.41
3/8	σ_c	21.0	26.7	27.6	28.0	28.3	28.5	28.6	28.7
	V	.69	.82	.90	1.00	1.08	1.27	1.43	1.61
1/2	σ_c	26.3	28.2	29.2	29.7	30.0	30.3	30.4	30.5
	V	.90	1.02	1.10	1.21	1.29	1.47	1.63	1.82
3/4	σ_c	29.5	30.0	30.5	30.8	31.3	31.5	31.7	31.8
	V	1.30	1.43	1.51	1.61	1.69	1.88	20.4	2.22

a/b = .3 — STIFFENER

Plate Thickness		A	B	C	D	E	F	G	H
1/4	σ_c	7.2	8.8	9.1	9.3	9.4	9.5	9.6	9.6
	V	.69	.82	.90	1.00	1.08	1.27	1.43	1.61
3/8	σ_c	11.2	16.2	18.4	19.6	20.3	20.8	21.2	21.4
	V	1.00	1.13	1.20	1.31	1.39	1.57	1.73	1.92
1/2	σ_c	15.3	22.5	25.7	26.4	26.9	27.3	27.5	27.6
	V	1.30	1.43	1.51	1.61	1.69	1.88	2.04	2.22
3/4	σ_c	26.0	27.2	28.2	28.9	29.5	30.0	30.3	30.5
	V	1.91	2.04	2.11	2.22	2.30	2.48	2.64	2.83

TABLE VIII

Results - Buckling Stress versus Sub-Panel Weight

σ_c = lb/in^2 x 10^{-3} V - in^3 x 10^{-3} b = 90"

a/b = .4 STIFFENER

Plate Thickness		A	B	C	D	E	F	G	H
1/4	σ_c	4.5	5.2	5.4	5.5	5.5	5.5	5.6	5.6
	V	.90	1.02	1.10	1.21	1.29	1.47	1.63	1.82
3/8	σ_c	7.6	10.2	11.2	11.7	12.0	12.2	12.4	12.5
	V	1.30	1.43	1.51	1.61	1.69	1.88	2.04	2.22
1/2	σ_c	10.4	15.1	17.6	19.0	20.3	21.1	21.7	21.9
	V	1.71	1.83	1.91	2.02	2.10	2.28	2.44	2.63
3/4	σ_c	18.2	22.9	25.9	26.7	27.5	28.1	28.5	28.6
	V	2.52	2.64	2.72	2.83	2.91	3.09	3.25	3.44

a/b = .5 STIFFENER

Plate Thickness		A	B	C	D	E	F	G	H
1/4	σ_c	3.2	3.6	3.6	3.7	3.7	3.7	3.8	3.8
	V	1.10	1.23	1.30	1.41	1.49	1.67	1.83	2.02
3/8	σ_c	5.5	7.2	7.7	8.0	8.2	8.3	8.4	8.4
	V	1.61	1.73	1.81	1.92	2.00	2.18	2.34	2.53
1/2	σ_c	7.7	10.7	12.3	13.2	13.8	14.3	14.6	14.8
	V	2.11	2.24	2.32	2.42	2.50	2.69	2.84	3.03
3/4	σ_c	13.6	17.3	20.6	23.6	25.4	26.1	26.6	26.8
	V	3.13	3.25	3.33	3.44	3.52	3.70	3.86	4.05

a/b = .8 STIFFENER

Plate Thickness		A	B	C	D	E	F	G	H
1/4	σ_c	1.6	1.7	1.8	1.8	1.8	1.8	1.8	1.8
	V	1.71	1.83	1.91	2.02	2.10	2.28	2.44	2.63
3/8	σ_c	3.2	3.7	3.9	3.9	4.0	4.0	4.0	4.0
	V	2.52	2.64	2.72	2.83	2.91	3.09	3.25	3.44
1/2	σ_c	4.8	6.0	6.5	6.8	6.9	7.0	7.1	7.2
	V	3.33	3.45	3.53	3.64	3.72	3.90	4.06	4.25
3/4	σ_c	9.0	10.9	12.4	13.4	14.3	15.0	15.6	15.8
	V	4.95	5.07	5.15	5.26	5.34	5.52	5.68	5.87

TABLE IX

Results - Buckling Stress versus Sub-Panel Weight

σ_c = lb/in^2 x 10^{-3} V - in^3 x 10^{-3} b = 120"

a/b = .1

Plate Thickness		STIFFENER A	B	C	D	E	F	G	H
1/4	σ_c	21.9	27.0	27.7	28.2	28.3	28.4	28.5	28.6
	V	.47	.66	.75	.89	1.00	1.24	1.46	1.70
3/8	σ_c	27.4	29.1	29.7	30.2	30.7	30.8	30.9	31.0
	V	.65	.84	.93	1.07	1.18	1.42	1.64	1.88
1/2	σ_c	29.4	29.9	30.6	30.9	31.2	31.5	31.7	31.8
	V	.83	1.02	1.11	1.25	1.36	1.60	1.82	2.06
3/4	σ_c	31.3	31.4	31.5	31.6	31.8	32.0	32.2	32.3
	V	1.19	1.38	1.47	1.61	1.72	1.96	2.18	2.42

a/b = .2

Plate Thickness		STIFFENER A	B	C	D	E	F	G	H
1/4	σ_c	7.5	9.8	10.7	11.1	11.4	11.6	11.7	11.8
	V	.83	1.02	1.11	1.25	1.36	1.60	1.82	2.06
3/8	σ_c	10.8	16.2	19.6	21.6	23.3	24.7	25.1	25.3
	V	1.19	1.38	1.47	1.61	1.72	1.96	2.18	2.42
1/2	σ_c	15.8	21.6	25.5	26.6	27.4	28.0	28.3	28.5
	V	1.55	1.74	1.83	1.97	2.08	2.32	2.54	2.78
3/4	σ_c	26.9	27.3	28.1	28.7	29.4	30.1	30.5	30.7
	V	2.27	2.46	2.55	2.69	2.80	3.04	3.26	3.50

a/b = .3

Plate Thickness		STIFFENER A	B	C	D	E	F	G	H
1/4	σ_c	3.8	4.6	5.1	5.2	5.3	5.3	5.4	5.4
	V	1.19	1.38	1.47	1.61	1.72	1.96	2.18	2.42
3/8	σ_c	6.0	8.7	10.2	11.0	11.7	12.2	12.5	12.7
	V	1.73	1.92	2.01	2.15	2.26	2.50	2.72	2.96
1/2	σ_c	8.1	11.5	14.2	16.0	17.8	19.2	20.3	20.8
	V	2.27	2.46	2.55	2.69	2.80	3.04	3.26	3.50
3/4	σ_c	15.9	18.2	21.7	25.1	26.2	27.2	28.0	28.3
	V	3.35	3.54	3.63	3.77	3.88	4.12	4.34	4.58

TABLE X

Results - Buckling Stress versus Sub-Panel Weight

σ_c = lb/in^2 x 10^{-3} V - in^3 x 10^{-3} b = 120"

a/b = .4 STIFFENER

Plate Thickness		A	B	C	D	E	F	G	H
1/4	σ_c	2.4	2.8	3.0	3.0	3.1	3.1	3.1	3.1
	V	1.55	1.74	1.83	1.97	2.08	2.32	2.54	2.18
3/8	σ_c	3.8	5.3	6.0	6.3	6.6	6.8	6.9	7.0
	V	2.27	2.46	2.55	2.69	2.80	3.04	3.26	3.50
1/2	σ_c	5.3	7.5	9.0	10.0	10.8	11.5	12.0	12.2
	V	2.99	3.18	3.27	3.41	3.52	3.76	3.98	4.22
3/4	σ_c	10.0	12.0	14.3	16.4	19.2	21.8	24.4	25.2
	V	4.43	4.62	4.71	4.85	4.96	5.20	5.42	5.56

a/b = .5 STIFFENER

Plate Thickness		A	B	C	D	E	F	G	H
1/4	σ_c	1.7	2.0	2.0	2.1	2.1	2.1	2.1	2.1
	V	1.91	2.10	2.19	2.33	2.44	2.68	2.90	3.14
3/8	σ_c	2.8	3.8	4.2	4.4	4.5	4.6	4.7	4.7
	V	2.81	3.00	3.09	3.23	3.34	3.58	3.80	4.04
1/2	σ_c	4.0	5.6	6.6	7.1	7.6	7.9	8.2	8.3
	V	3.71	3.90	3.99	4.13	4.24	4.48	4.70	4.94
3/4	σ_c	7.4	9.1	10.7	12.2	13.9	15.5	17.0	17.7
	V	5.51	5.70	5.79	5.93	6.04	6.28	6.50	6.74

a/b = .8 STIFFENER

Plate Thickness		A	B	C	D	E	F	G	H
1/4	σ_c	.9	1.0	1.0	1.0	1.0	1.0	1.0	1.0
	V	2.99	3.18	3.27	3.41	3.52	3.76	3.98	4.22
3/8	σ_c	1.7	2.0	2.2	2.2	2.2	2.2	2.3	2.3
	V	4.43	4.62	4.71	4.85	4.96	5.20	5.42	5.66
1/2	σ_c	2.6	3.2	3.6	3.7	3.8	3.9	4.0	4.0
	V	5.87	6.06	6.15	6.29	6.40	6.64	6.86	7.10
3/4	σ_c	4.9	5.8	6.6	7.2	7.8	8.2	8.6	8.8
	V	8.75	8.94	9.03	9.17	9.28	9.52	9.74	9.98

TABLE XI

Results - Buckling Stress versus Sub-Panel Weight

σ_c = lb/in^2 x 10^{-3} V - in^3 x 10^3 b = 180"

a/b = .1 STIFFENER

Plate Thickness		A	B	C	D	E	F	G	H
1/4	σ_c	8.6	13.0	15.6	17.0	18.2	19.1	19.9	20.2
	V	1.0	1.3	1.4	1.6	1.8	2.1	2.4	2.8
3/8	σ_c	14.4	19.3	24.0	25.9	26.9	27.6	28.0	28.3
	V	1.4	1.7	1.8	2.0	2.2	2.5	2.9	3.2
1/2	σ_c	24.4	25.6	26.8	27.6	28.4	29.2	29.8	30.1
	V	1.8	2.1	2.2	2.4	2.6	2.9	3.3	3.6
3/4	σ_c	29.0	29.4	29.5	29.7	30.0	30.4	31.0	31.2
	V	2.6	2.9	3.0	3.2	3.4	3.8	4.1	4.4

a/b = .2 STIFFENER

Plate Thickness		A	B	C	D	E	F	G	H
1/4	σ_c	2.7	4.0	4.5	4.8	5.0	5.1	5.2	5.2
	V	1.8	2.1	2.2	2.4	2.6	2.9	3.3	3.6
3/8	σ_c	4.3	6.2	7.8	8.8	9.8	10.6	11.2	11.4
	V	2.6	2.9	3.0	3.2	3.4	3.8	4.1	4.4
1/2	σ_c	6.6	8.5	10.5	12.2	14.4	16.5	18.2	19.2
	V	3.4	3.7	3.8	4.0	4.2	4.6	4.9	5.3
3/4	σ_c	14.4	14.9	16.5	18.2	21.4	25.2	26.7	27.4
	V	5.0	5.3	5.4	5.7	5.8	6.2	6.5	6.9

a/b = .3 STIFFENER

Plate Thickness		A	B	C	D	E	F	G	H
1/4	σ_c	1.5	2.0	2.2	2.2	2.3	2.4	2.4	2.4
	V	2.6	2.9	3.0	3.2	3.4	3.8	4.1	4.4
3/8	σ_c	2.3	3.3	4.0	4.4	4.8	5.0	5.2	5.3
	V	3.8	4.1	4.2	4.4	4.6	5.0	5.3	5.7
1/2	σ_c	3.4	4.5	5.6	6.4	7.2	8.0	8.8	9.0
	V	5.0	5.3	5.4	5.7	5.8	6.2	6.5	6.9
3/4	σ_c	6.9	7.6	8.6	9.7	11.4	13.5	16.3	17.8
	V	7.5	7.7	7.9	8.1	8.2	8.6	8.9	9.3

TABLE XII

Results - Buckling Stress versus Sub-Panel Weight

σ_c = lb/in^2 x 10^{-3} V - in^3 x 10^{-3} b = 180"

a/b = .4

Plate Thickness		STIFFENER A	B	C	D	E	F	G	H
1/4	σ_c	0.9	1.2	1.3	1.3	1.4	1.4	1.4	1.4
	V	3.4	3.7	3.8	4.0	4.2	4.6	4.9	5.3
3/8	σ_c	1.5	2.1	2.5	2.7	2.9	3.0	3.0	3.1
	V	5.0	5.3	5.4	5.7	5.8	6.2	6.5	6.9
1/2	σ_c	2.2	2.9	3.6	4.0	4.5	4.9	5.2	5.3
	V	6.7	6.9	7.1	7.3	7.4	7.8	8.1	8.5
3/4	σ_c	4.4	4.9	5.7	6.4	7.4	8.7	10.1	10.8
	V	9.9	10.2	10.3	10.5	10.7	11.0	11.4	11.7

a/b = .5

Plate Thickness		STIFFENER A	B	C	D	E	F	G	H
1/4	σ_c	0.7	0.8	0.9	0.9	0.9	0.9	0.9	0.9
	V	4.2	4.5	4.6	4.8	5.0	5.4	5.7	6.1
3/8	σ_c	1.1	1.6	1.8	1.9	2.0	2.0	2.1	2.1
	V	6.2	6.5	6.7	6.9	7.0	7.4	7.7	8.1
1/2	σ_c	1.62	2.2	2.7	2.9	3.2	3.4	3.6	3.6
	V	8.2	8.5	8.7	8.9	9.1	9.4	9.7	10.1
3/4	σ_c	3.2	3.7	4.2	4.8	5.5	6.4	7.2	7.5
	V	12.3	12.6	12.7	12.9	13.1	13.5	13.8	14.2

a/b = .8

Plate Thickness		STIFFENER A	B	C	D	E	F	G	H
1/4	σ_c	0.4	0.4	0.4	0.4	0.4	0.4	0.4	0.5
	V	6.7	6.9	7.1	7.3	7.4	7.8	8.1	8.5
3/8	σ_c	0.7	0.9	0.9	1.0	1.0	1.0	1.0	1.0
	V	9.9	10.2	10.3	10.5	10.7	11.0	11.3	11.7
1/2	σ_c	1.1	1.3	1.5	1.6	1.7	1.7	1.8	1.8
	V	13.1	13.4	13.6	13.8	13.9	14.3	14.6	15.0
3/4	σ_c	2.1	2.4	2.7	3.0	3.2	3.5	3.7	3.8
	V	19.6	19.9	20.0	20.2	20.4	20.8	21.1	21.5

TABLE XIII

Results of Buckling Stress versus Sub-Panel Weight

σ_c = lb/in^2 x 10^{-3} V - in^3 x 10^{-3} b = 250"

a/b = .1 STIFFENER

Plate Thickness		A	B	C	D	E	F	G	H
1/4	σ_c	4.0	6.0	7.4	8.3	9.0	9.6	10.1	10.3
	V	1.8	2.2	2.4	2.7	2.9	3.4	3.8	4.4
3/8	σ_c	7.3	9.1	11.1	13.0	15.5	17.9	20.3	21.4
	V	2.6	3.0	3.2	3.4	3.7	4.2	4.6	5.1
1/2	σ_c	12.6	13.4	15.2	17.3	20.2	24.5	25.4	27.0
	V	3.4	3.7	3.9	4.2	4.4	5.0	5.4	5.9
3/4	σ_c	25.5	25.9	26.0	26.2	26.8	27.6	28.6	29.2
	V	4.9	5.3	5.5	5.8	6.0	6.5	7.0	7.5

a/b = .2 STIFFENER

Plate Thickness		A	B	C	D	E	F	G	H
1/4	σ_c	1.3	1.9	2.2	2.4	2.5	2.6	2.7	2.7
	V	3.4	3.7	3.9	4.2	4.4	5.0	5.4	5.9
3/8	σ_c	2.1	2.9	3.6	4.2	4.8	5.3	5.7	5.9
	V	4.9	5.3	5.5	5.8	6.0	6.5	7.0	7.5
1/2	σ_c	3.4	4.0	4.8	5.6	6.6	7.8	9.0	9.6
	V	6.5	6.9	7.1	7.4	7.6	8.1	8.5	9.0
3/4	σ_c	6.7	7.4	8.2	8.8	10.4	11.8	14.9	16.9
	V	9.6	10.0	10.2	10.5	10.7	11.2	11.7	12.1

a/b = .3 STIFFENER

Plate Thickness		A	B	C	D	E	F	G	H
1/4	σ_c	0.7	1.0	1.1	1.1	1.2	1.2	1.2	1.2
	V	4.9	5.3	5.5	5.8	6.0	6.5	7.0	7.5
3/8	σ_c	1.1	1.6	1.9	2.1	2.4	2.5	2.6	2.7
	V	7.3	7.7	7.8	8.1	8.4	8.9	9.3	9.8
1/2	σ_c	1.6	2.1	2.6	3.1	3.4	3.9	4.4	4.6
	V	9.6	10.0	10.2	10.5	10.7	11.2	11.7	12.2
3/4	σ_c	3.5	3.7	4.1	4.6	5.3	6.4	7.8	8.6
	V	14.3	14.7	14.9	15.2	15.4	15.9	16.4	16.9

TABLE XIV

Results - Buckling Stress versus Sub-Panel Weight

σ_c = lb/in^2 x 10^{-3} V - in^3 x 10^{-3} b = 250"

a/b = .4		STIFFENER							
Plate Thickness		A	B	C	D	E	F	G	H
1/4	σ_c	0.4	0.6	0.7	0.7	0.7	0.7	0.7	0.7
	V	6.5	6.9	7.1	7.4	7.6	8.1	8.5	9.0
3/8	σ_c	0.7	1.0	1.2	1.3	1.4	1.5	1.6	1.6
	V	9.6	10.0	10.2	10.5	10.7	11.2	11.7	12.2
1/2	σ_c	1.1	1.4	1.7	1.9	2.2	2.4	2.6	2.7
	V	12.7	13.1	13.3	13.6	13.8	14.3	14.8	15.3
3/4	σ_c	2.2	2.4	2.7	3.0	3.4	4.0	4.8	5.2
	V	19.0	19.4	19.6	19.9	20.1	20.6	21.0	21.6

a/b = .5		STIFFENER							
Plate Thickness		A	B	C	D	E	F	G	H
1/4	σ_c	0.3	0.4	0.5	0.5	0.5	0.5	0.5	0.5
	V	8.0	8.4	8.6	8.9	9.1	9.6	10.1	10.6
3/8	σ_c	0.6	0.8	0.9	0.9	1.0	1.0	1.1	1.1
	V	11.9	12.2	12.4	12.7	13.0	13.5	13.9	14.4
1/2	σ_c	0.8	1.0	1.3	1.4	1.6	1.7	1.8	1.9
	V	15.9	16.2	16.4	16.7	16.9	17.5	17.9	18.4
3/4	σ_c	1.6	1.8	2.0	2.3	2.6	3.0	3.5	3.8
	V	23.7	24.1	24.3	24.6	24.8	25.3	25.7	26.2

a/b = .8		STIFFENER							
Plate Thickness		A	B	C	D	E	F	G	H
1/4	σ_c	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2
	V	12.7	13.1	13.3	13.6	13.8	14.3	14.8	15.3
3/8	σ_c	0.3	0.4	0.5	0.5	0.5	0.5	0.5	0.5
	V	19.0	19.4	19.6	19.9	20.1	20.6	21.0	21.5
1/2	σ_c	0.5	0.7	0.7	0.8	0.8	0.9	0.9	0.9
	V	25.2	25.6	25.8	26.1	26.3	26.8	27.3	27.8
3/4	σ_c	1.1	1.2	1.3	1.4	1.6	1.7	1.9	2.0
	V	37.7	38.1	38.3	38.6	38.8	39.3	39.8	40.3

TABLE XV

Results - Buckling Stress versus Sub-Panel Weight

σ_c = lb/in^2 x 10^{-3} V - in^3 x 10^{-3} b = 360"

a/b = .1 STIFFENER

Plate Thickness		A	B	C	D	E	F	G	H
1/4	σ_c	1.8	2.5	3.2	3.7	4.1	4.5	4.8	4.9
	V	3.6	4.1	4.4	4.8	5.2	5.9	6.5	7.3
3/8	σ_c	3.5	4.0	4.8	5.6	6.6	7.8	9.2	9.9
	V	5.2	5.8	6.0	6.4	6.8	7.5	8.1	8.9
1/2	σ_c	5.9	6.2	6.8	7.5	8.7	10.4	13.1	14.9
	V	6.8	7.4	7.6	8.1	8.4	9.1	9.8	10.5
3/4	σ_c	12.6	13.4	13.7	14.0	14.5	16.1	19.4	22.3
	V	10.1	10.6	10.9	11.3	11.6	12.4	13.0	13.7

a/b = .2 STIFFENER

Plate Thickness		A	B	C	D	E	F	G	H
1/4	σ_c	0.6	0.8	1.0	1.1	1.2	1.2	1.3	1.3
	V	6.8	7.4	7.6	8.1	8.4	9.1	9.8	10.5
3/8	σ_c	1.0	1.2	1.5	1.8	2.1	2.4	2.6	2.7
	V	10.0	10.6	10.9	11.3	11.6	12.4	13.0	13.7
1/2	σ_c	1.6	1.8	2.1	2.4	2.8	3.5	4.0	4.4
	V	13.3	13.9	14.1	14.6	14.9	15.6	16.2	17.0
3/4	σ_c	3.4	3.6	3.7	3.8	4.3	5.0	6.3	7.2
	V	19.8	20.3	20.6	21.0	21.4	22.1	22.7	23.5

a/b = .3 STIFFENER

Plate Thickness		A	B	C	D	E	F	G	H
1/4	σ_c	0.3	0.4	0.5	0.5	0.6	0.6	0.6	0.6
	V	10.1	10.6	10.9	11.3	11.6	12.4	13.0	13.7
3/8	σ_c	0.5	0.7	0.8	0.9	1.1	1.2	1.2	1.3
	V	14.9	15.5	15.7	16.2	16.5	17.2	17.9	18.6
1/2	σ_c	0.8	0.9	1.1	1.3	1.5	1.7	1.9	2.0
	V	19.8	20.3	20.6	21.0	21.4	22.1	22.7	23.5
3/4	σ_c	1.7	1.8	1.9	2.0	2.3	2.7	3.4	3.8
	V	29.5	30.1	30.3	30.8	31.1	31.8	32.4	33.2

TABLE XVI

Results - Buckling Stress versus Sub-Panel Weight

$\sigma_c = lb/in^2 \times 10^{-3}$ V - $in^3 \times 10^{-3}$ b = 360"

a/b = .4 STIFFENER

Plate Thickness		A	B	C	D	E	F	G	H
1/4	σ_c	0.2	0.3	0.3	0.3	0.3	0.3	0.3	0.3
	V	13.3	13.9	14.1	14.5	14.9	15.6	16.2	17.0
3/8	σ_c	0.3	0.4	0.5	0.6	0.6	0.7	0.7	0.8
	V	19.8	20.3	20.6	21.0	21.4	22.1	22.7	23.5
1/2	σ_c	0.5	0.6	0.7	0.8	1.0	1.1	1.2	1.3
	V	26.3	26.8	27.1	27.5	27.8	28.6	29.2	29.9
3/4	σ_c	1.1	1.1	1.2	1.3	1.5	1.8	2.2	2.4
	V	39.2	39.8	40.0	40.5	40.8	41.5	42.2	42.9

a/b = .5 STIFFENER

Plate Thickness		A	B	C	D	E	F	G	H
1/4	σ_c	0.1	0.2	0.2	0.2	0.2	0.2	0.2	0.2
	V	16.5	17.1	17.4	17.8	18.1	18.8	19.5	20.2
3/8	σ_c	0.2	0.3	0.4	0.4	0.5	0.5	0.5	0.5
	V	24.6	25.2	25.5	25.9	26.2	26.9	27.6	28.3
1/2	σ_c	0.4	0.5	0.5	0.6	0.7	0.8	0.8	0.9
	V	32.7	33.3	33.6	34.0	34.3	35.0	35.7	36.4
3/4	σ_c	0.8	0.8	0.9	1.0	1.1	1.3	1.6	1.7
	V	48.9	49.5	49.8	50.2	50.5	51.2	51.9	52.6

a/b = .8 STIFFENER

Plate Thickness		A	B	C	D	E	F	G	H
1/4	σ_c	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1
	V	26.2	26.8	27.1	27.5	27.8	28.5	29.2	29.9
3/8	σ_c	0.2	0.2	0.2	0.2	0.2	0.2	0.3	0.3
	V	39.2	39.8	40.1	40.5	40.8	41.5	42.2	42.9
1/2	σ_c	0.3	0.4	0.4	0.4	0.4	0.4	0.4	0.4
	V	52.2	52.8	53.1	53.5	53.8	54.5	55.2	55.9
3/4	σ_c	0.5	0.6	0.7	0.8	0.9	0.9	1.0	1.0
	V	78.1	78.7	79.0	79.4	79.7	80.4	81.1	81.8

FIGURE I

Sub-Panel Critical Buckling Strength

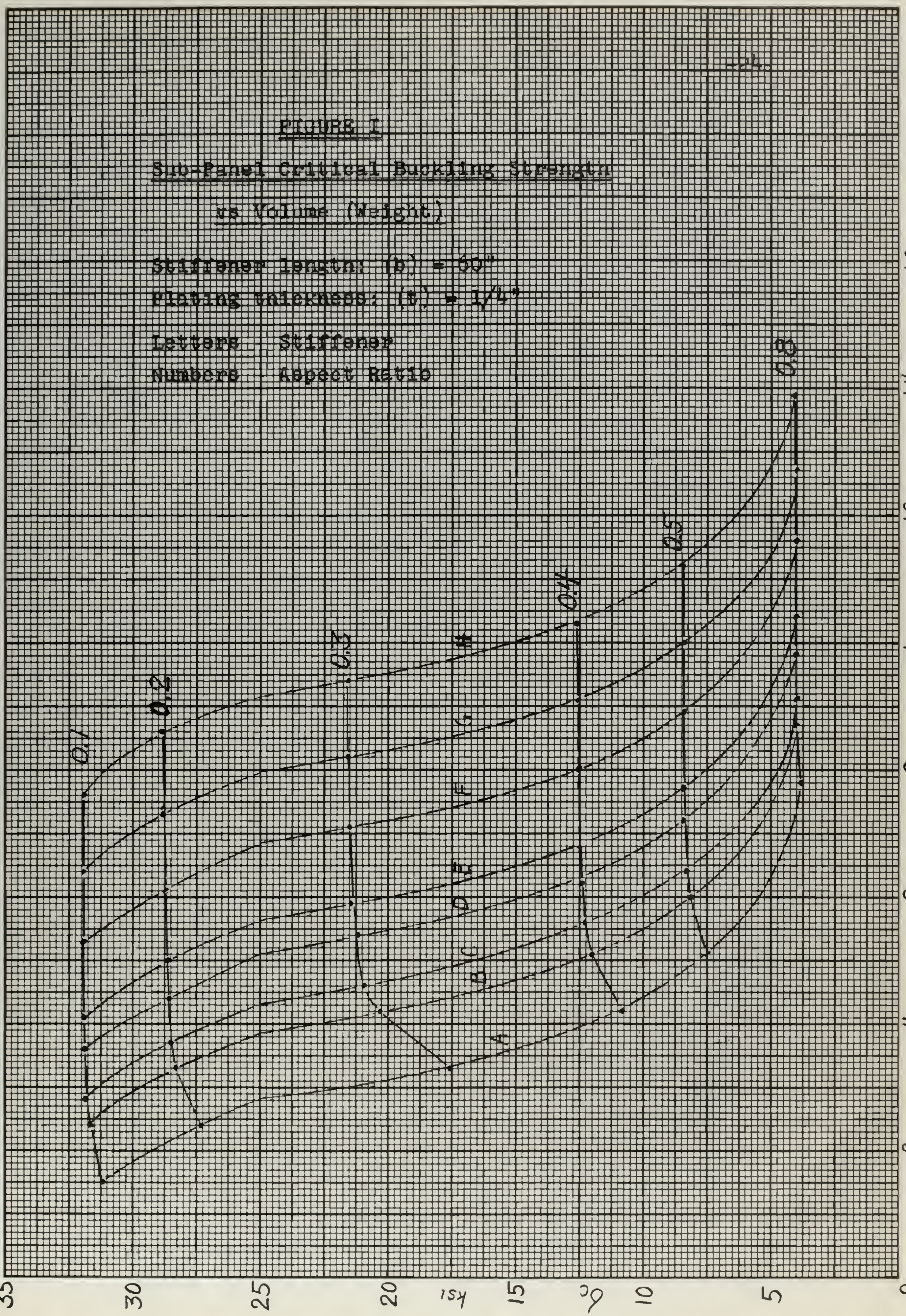
vs Volume (Weight)

Stiffener length: (b) = 60"

Plating thickness: (t) = 1/4"

Letters - Stiffener

Numbers - Aspect Ratio



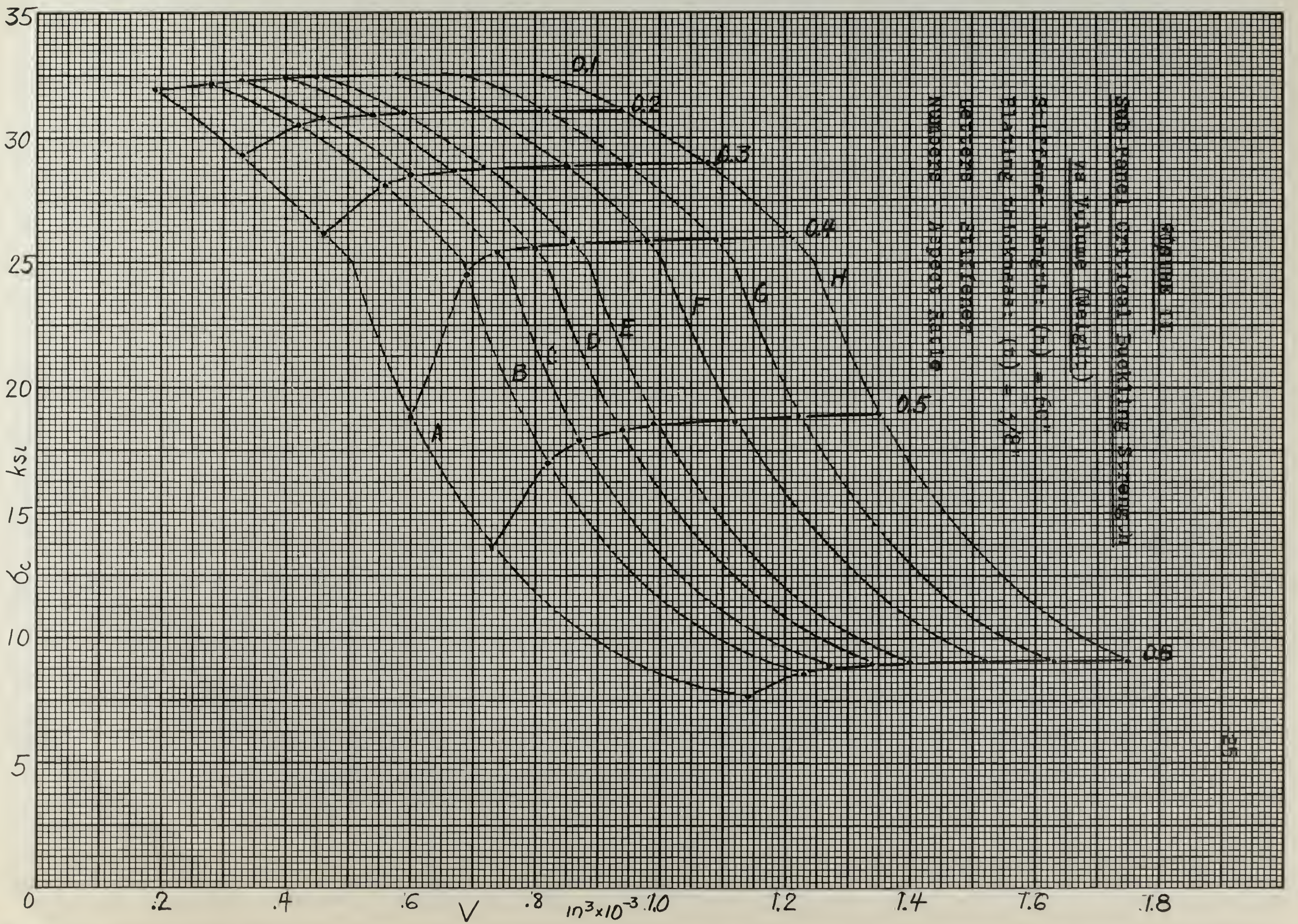


FIGURE II
Sub-Panel Critical Buckling Strength
vs Volume (Weight)
Stiffener length: (h) = 60"
Plating thickness: (t) = 1/8"
Letters - Stiffener
Numbers - Aspect Ratio
0.1
0.2
0.3
0.4
0.5
0.6
A
B
C
D
E
F
G
H
σ_c ksi
V in^3×10^{-3}
0
.2
.4
.6
.8
1.0
1.2
1.4
1.6
1.8
5
10
15
20
25
30
35

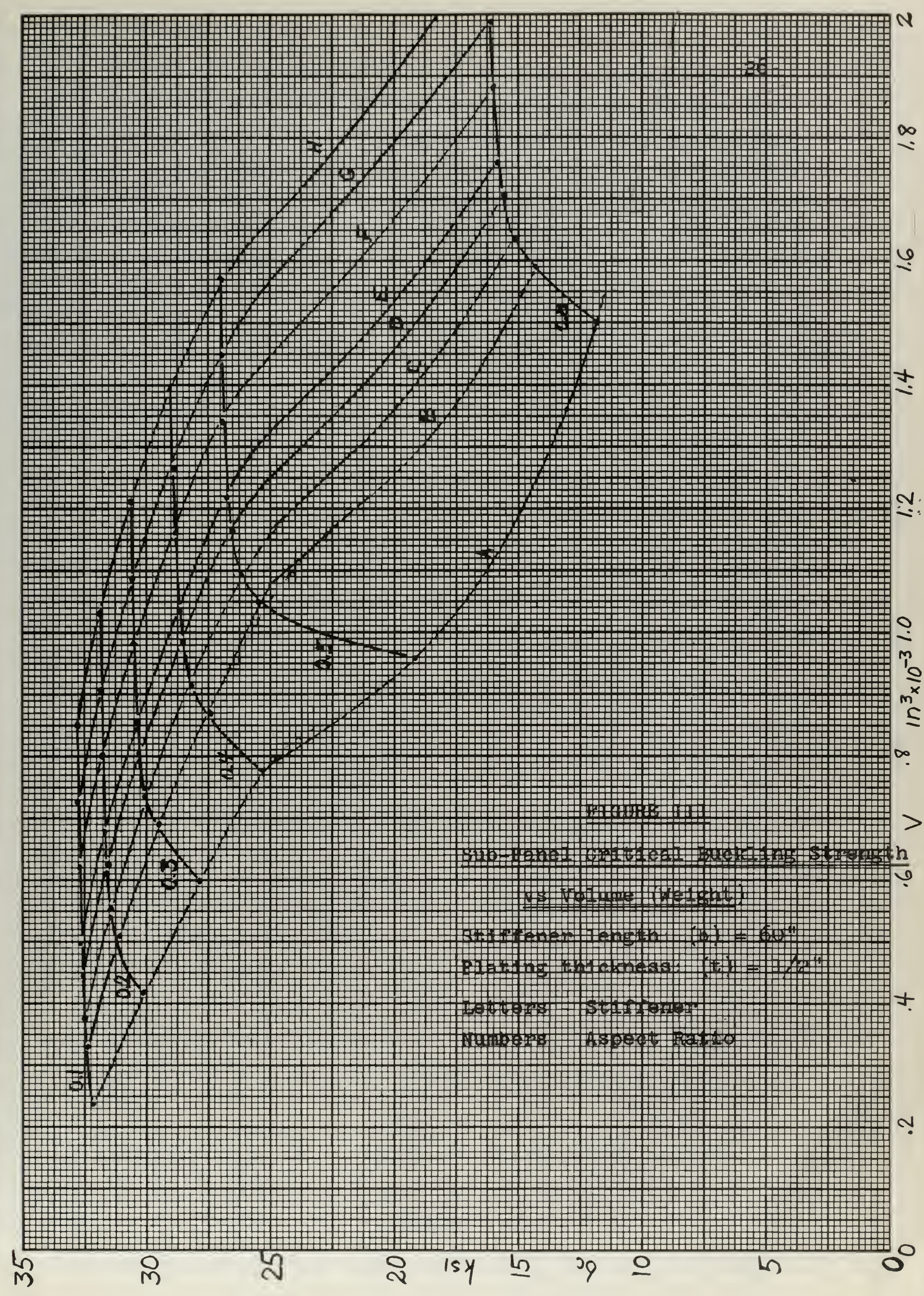


FIGURE III

Sub-Panel Critical Buckling Strength

vs Volume (Weight)

Stiffener length (b) = 60"

Plating thickness (t) = 1/2"

Letters Stiffener

Numbers Aspect Ratio

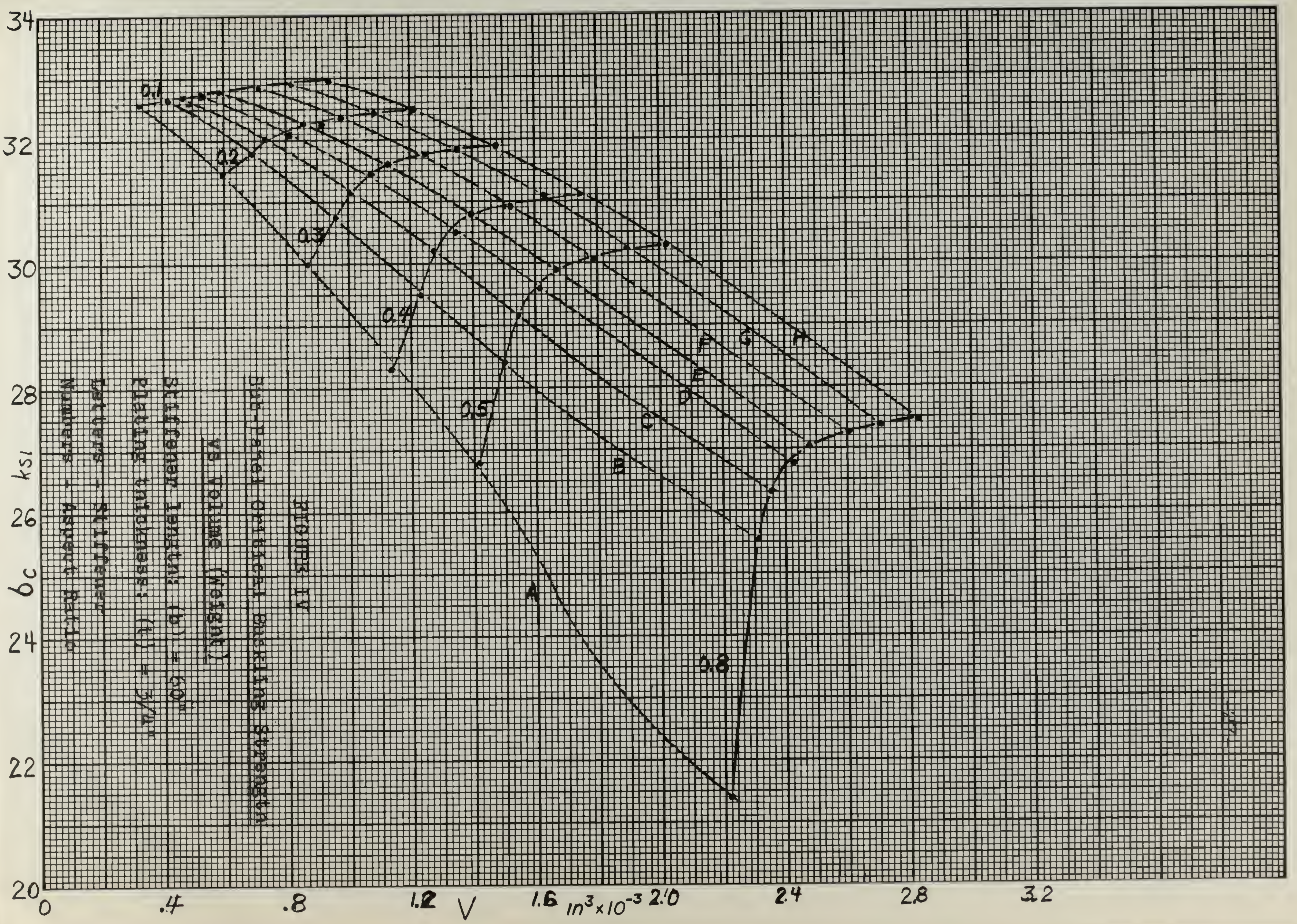


FIGURE IV

Sub-Panel Critical Buckling Strength

vs Volume (Weight)

Stiffener length: (b) = 60"

Plating thickness: (t) = 3/4"

Letters - Stiffener

Numbers - Aspect Ratio

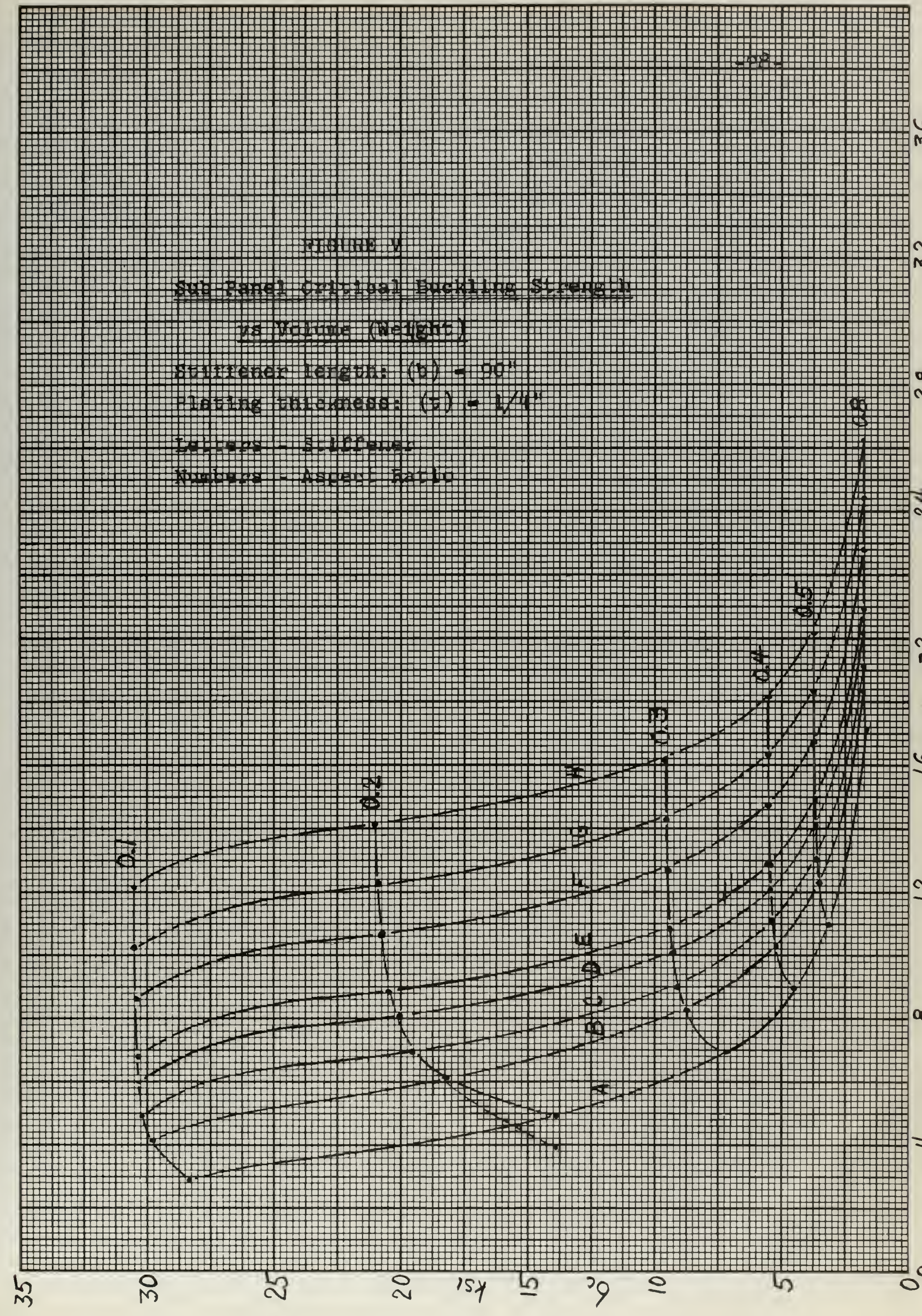


FIGURE V

Sub-Panel Critical Buckling Strength vs Volume (Weight)

Stiffener length: (b) = 90"
Plating thickness: (t) = 1/4"

Letters - Stiffener
Numbers - Aspect Ratio

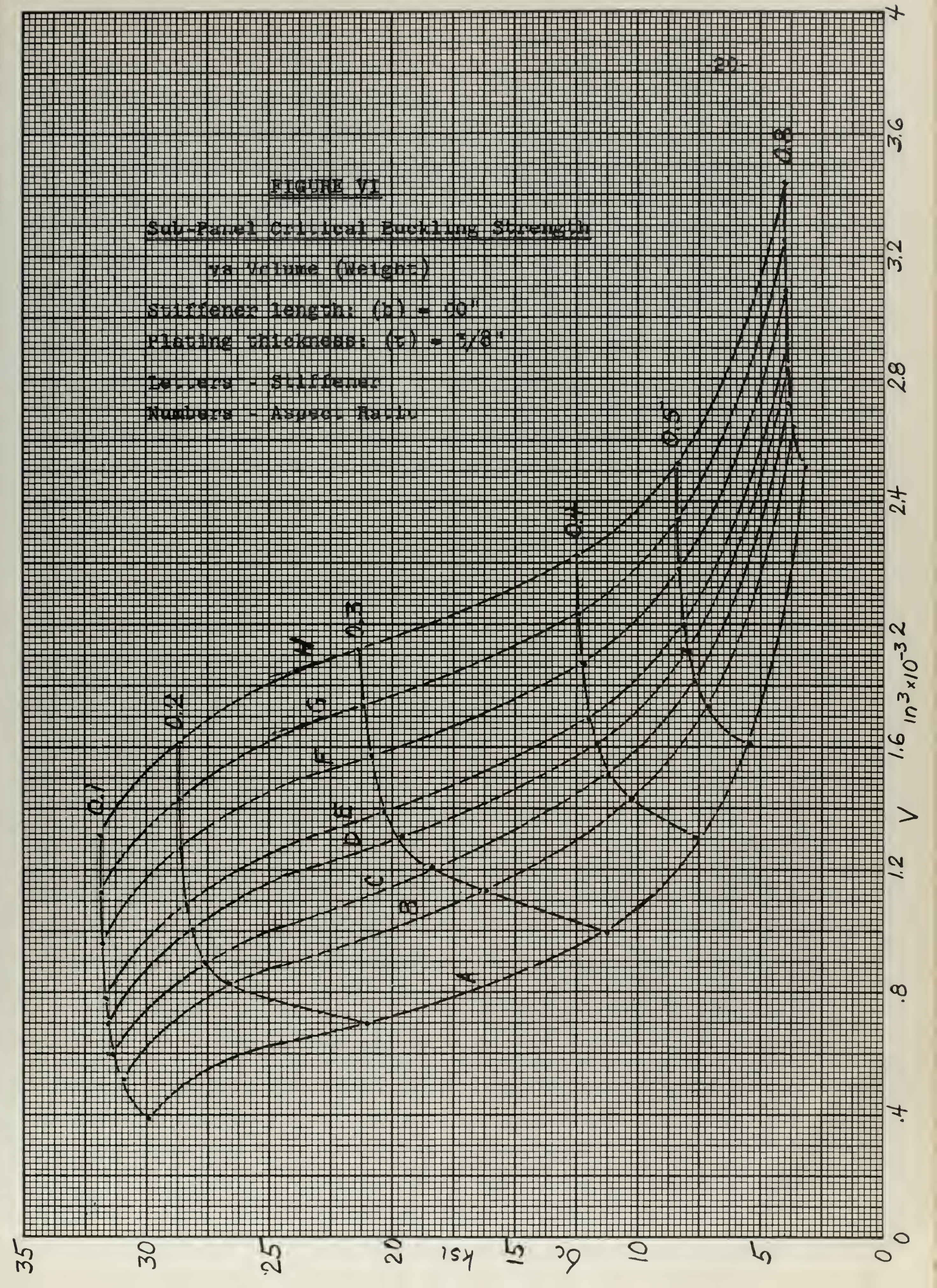


FIGURE VI
Sub-Panel Critical Buckling Strength
vs Volume (Weight)
Stiffener length: (b) = 90"
Plating thickness: (t) = 5/8"
Letters - Stiffener
Numbers - Aspect Ratio
A
B
C
D
E
F
G
H
0.1
0.2
0.3
0.4
0.5
0.8
20
V $in^3 \times 10^{-3}$ 2
σ_c ksi
0
5
10
15
20
25
30
35
.4
.8
1.2
1.6
2.0
2.4
2.8
3.2
3.6
4

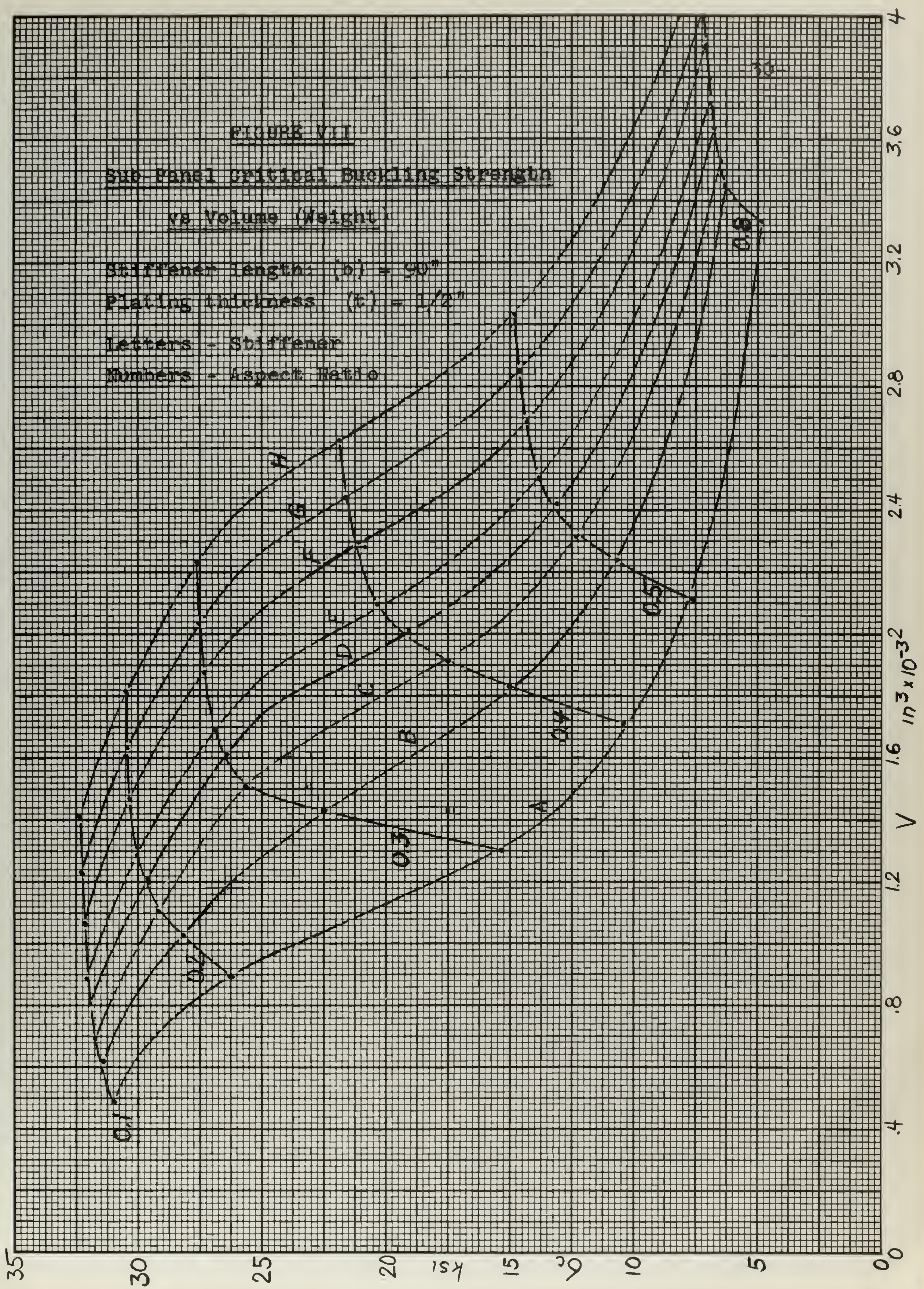


FIGURE VII

Sub-Panel Critical Buckling Strength

vs Volume (Weight)

Stiffener length (b) = 90"

Plating thickness (t) = 1/2"

Letters - Stiffener

Numbers - Aspect Ratio

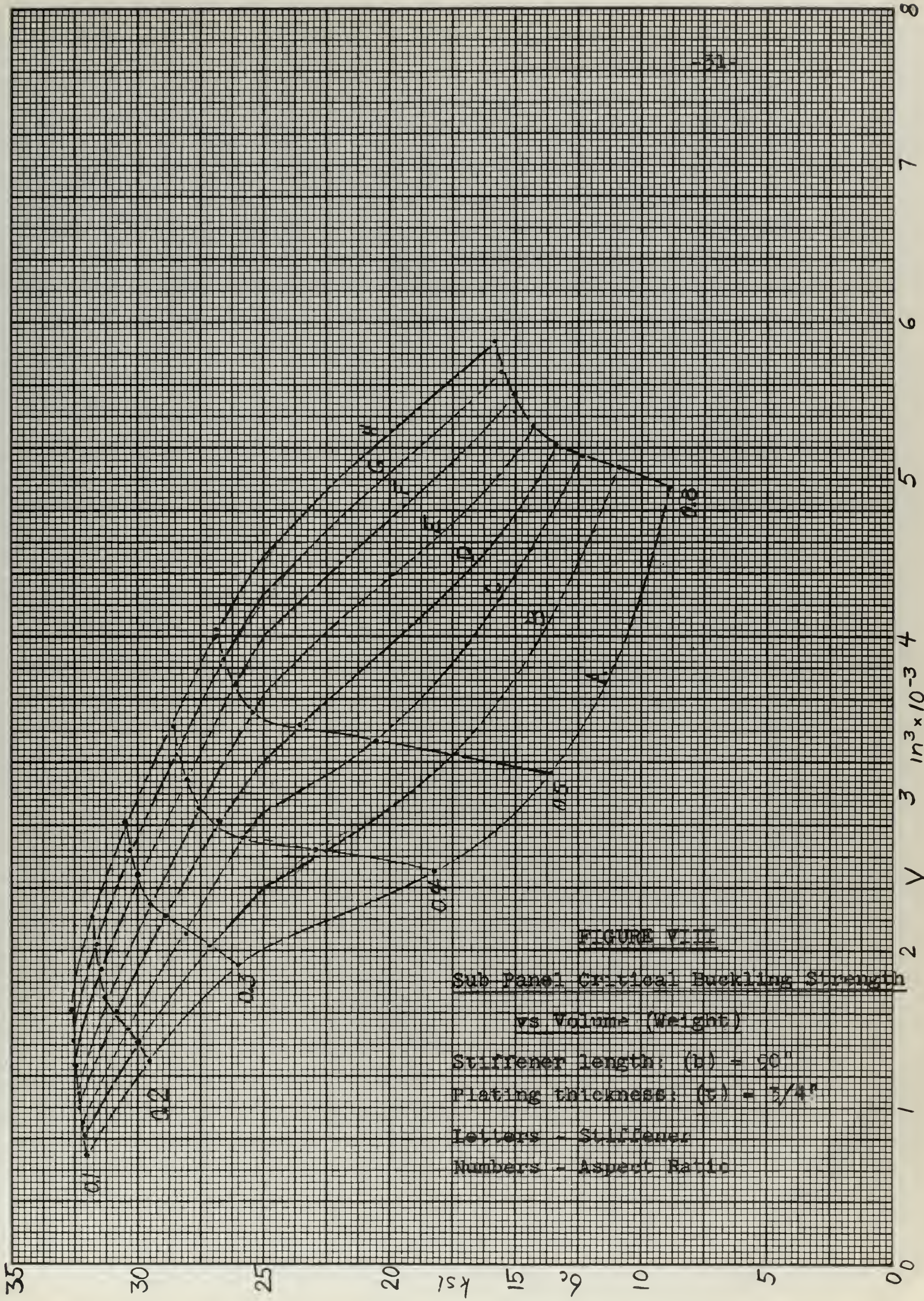


FIGURE VIII

Sub Panel Critical Buckling Strength

vs Volume (Weight)

Stiffener length: (b) = 90"

Plating thickness: (t) = 3/4"

Letters - Stiffener

Numbers - Aspect Ratio

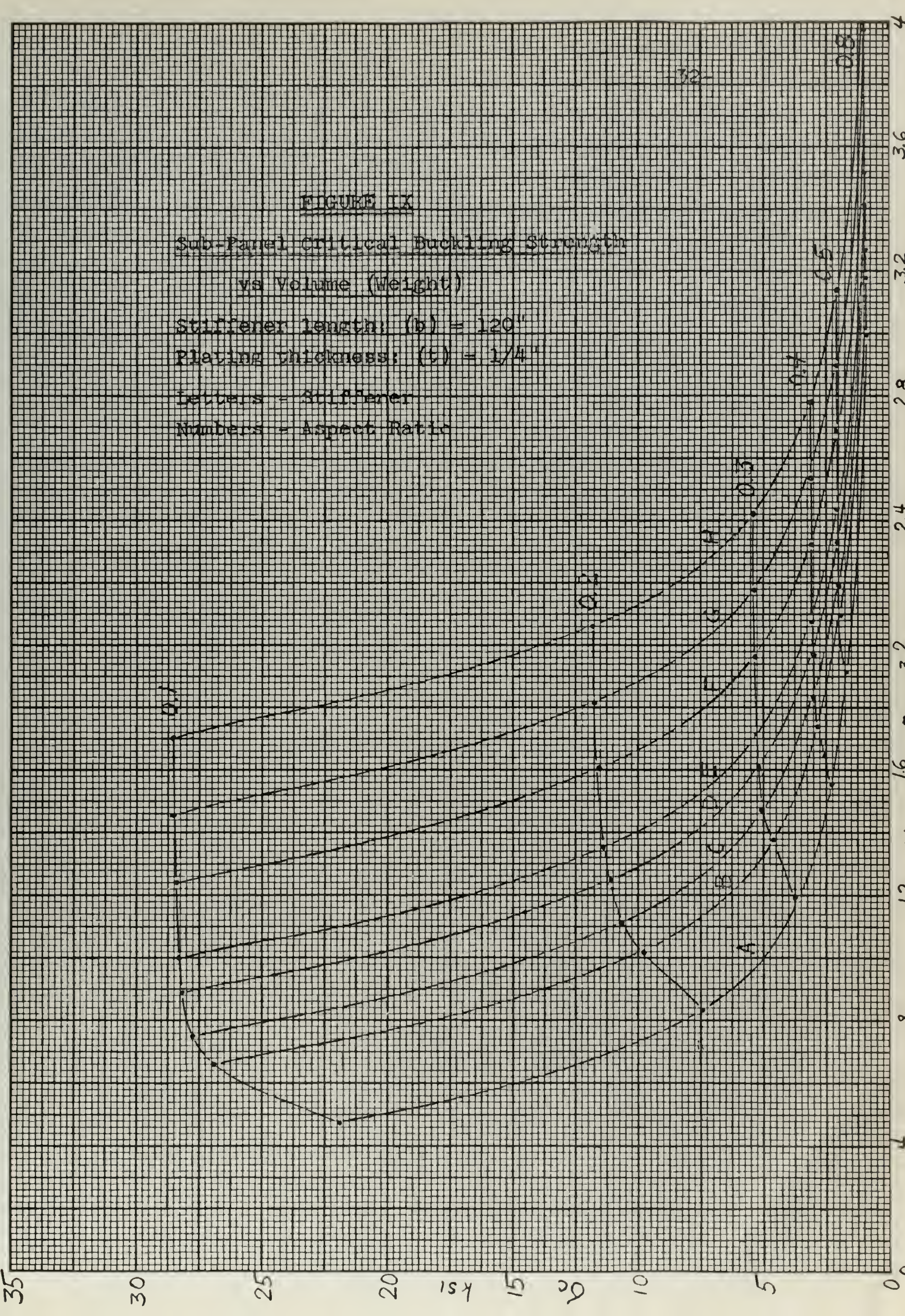


FIGURE IX
Sub-Panel Critical Buckling Strength
vs Volume (Weight)
Stiffener length: (b) = 120"
Plating thickness: (t) = 1/4"
Letters - Stiffener
Numbers - Aspect Ratio
0.1
0.2
0.3
0.4
0.5
0.8
A
B
C
D
E
F
G
H
V
in^3 x 10^{-3}
ksi
σ_c
0
.4
.8
1.2
1.6
2
2.4
2.8
3.2
3.6
4
5
10
15
20
25
30
35

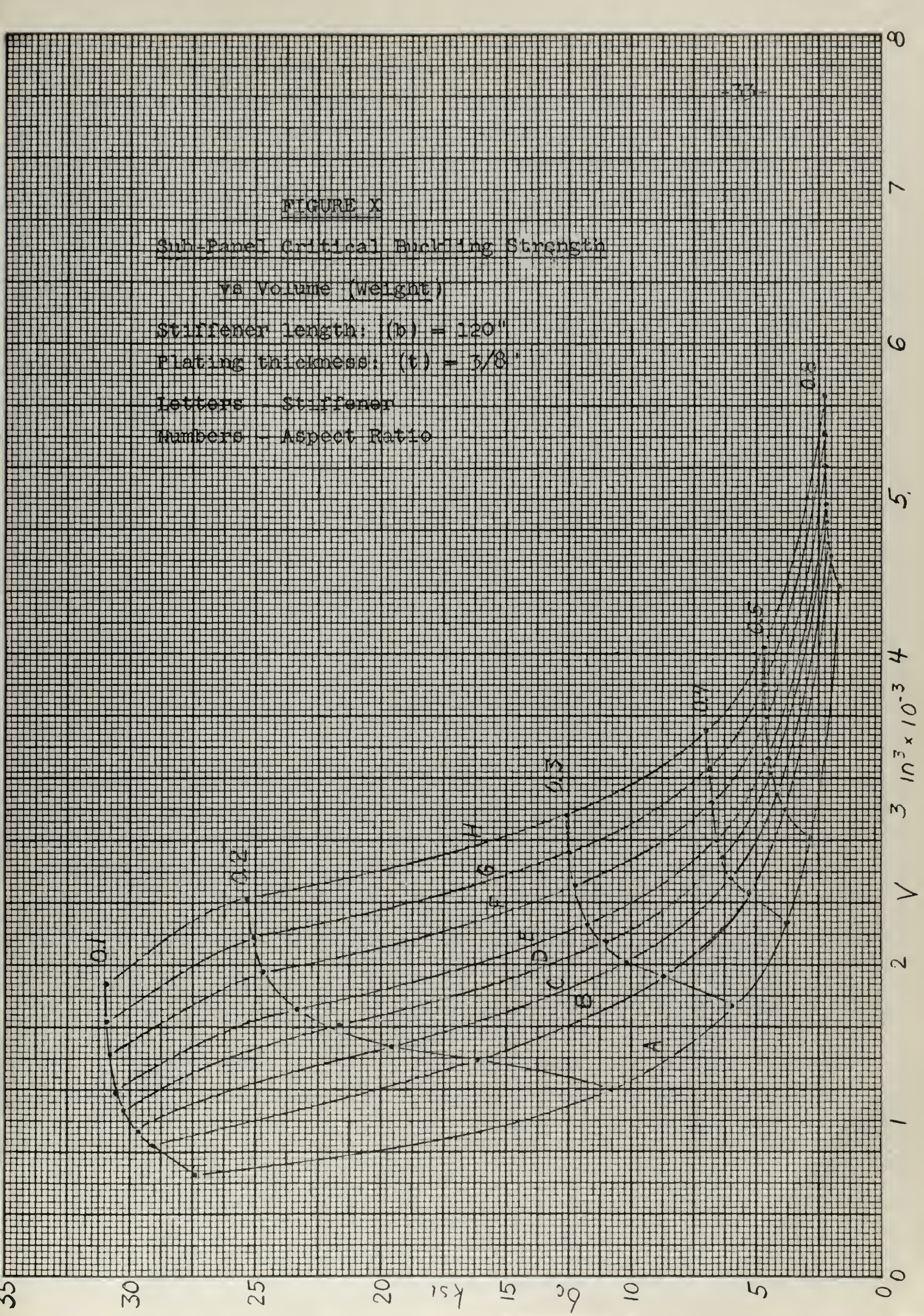


FIGURE X

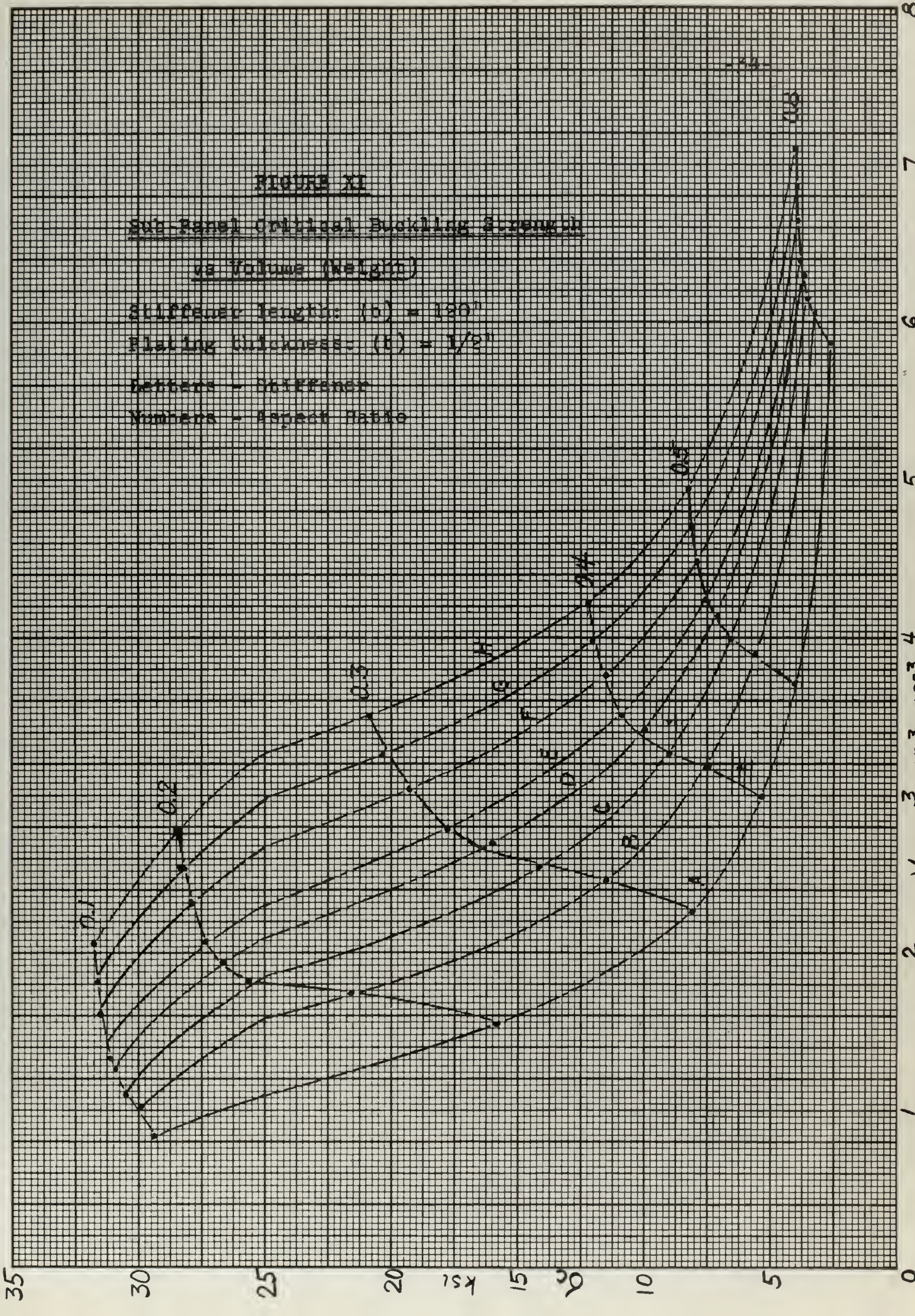
Sub-Panel Critical Buckling Strength vs Volume (Weight)

Stiffener length: (b) = 120"

Plating thickness: (t) = 3/8"

Letters - Stiffener

Numbers - Aspect Ratio


-64-
FIGURE XI
Sub-Panel Critical Buckling Strength
vs Volume (Weight)
Stiffener Length: (b) = 180"
Plating thickness: (t) = 1/2"
Letters - Stiffener
Numbers - Aspect Ratio
0.1
0.2
0.3
0.4
0.5
0.6
A
B
C
D
E
F
G
H
V in³ x 10⁻³
0
1
2
3
4
5
6
7
8
σ ksi
0
5
10
15
20
25
30
35

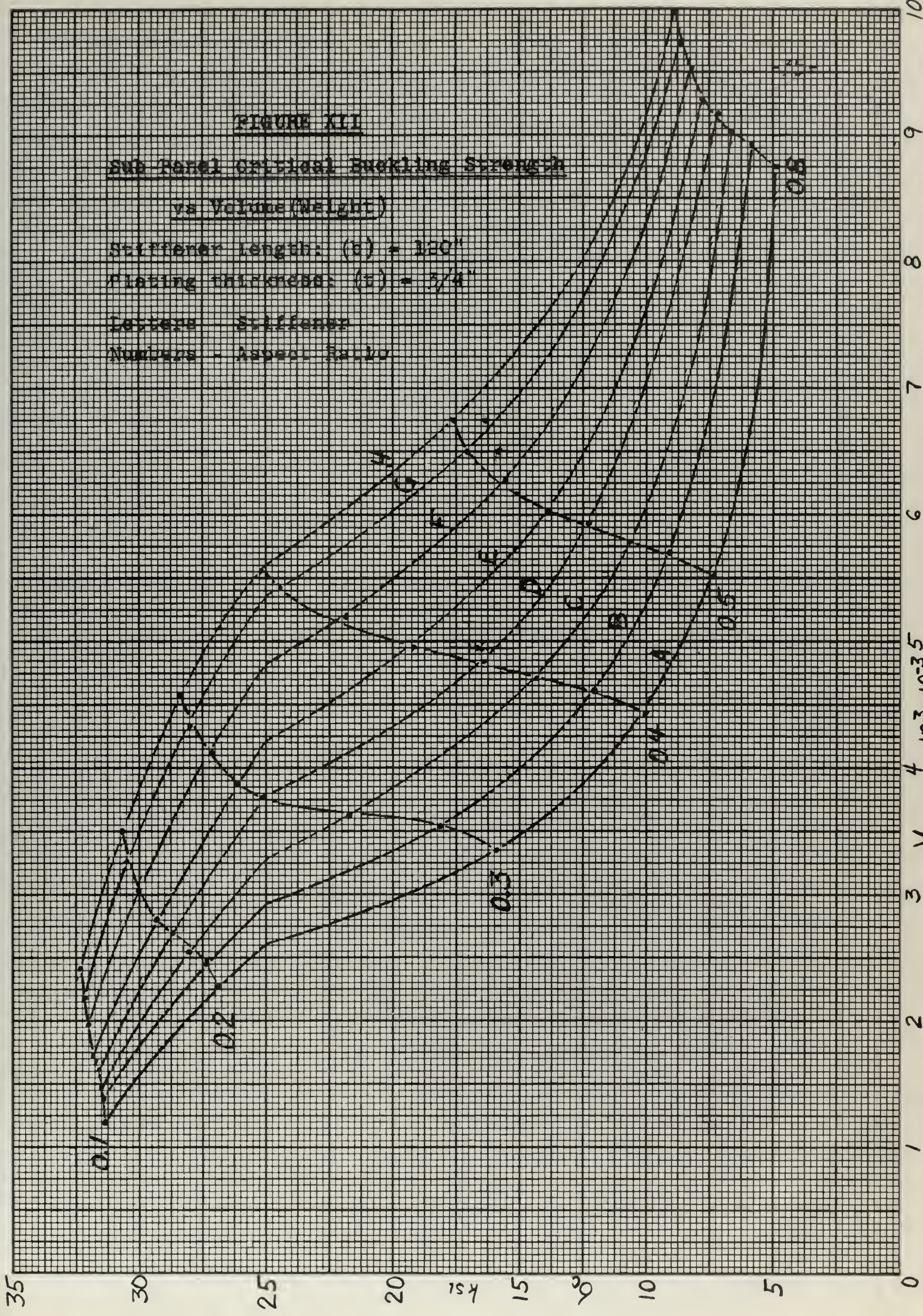


FIGURE XII

Sub-Panel Critical Buckling Strength

vs Volume(Weight)

Stiffener length: (b) = 120"

Plating thickness: (t) = 3/4"

Letters - Stiffener

Numbers - Aspect Ratio

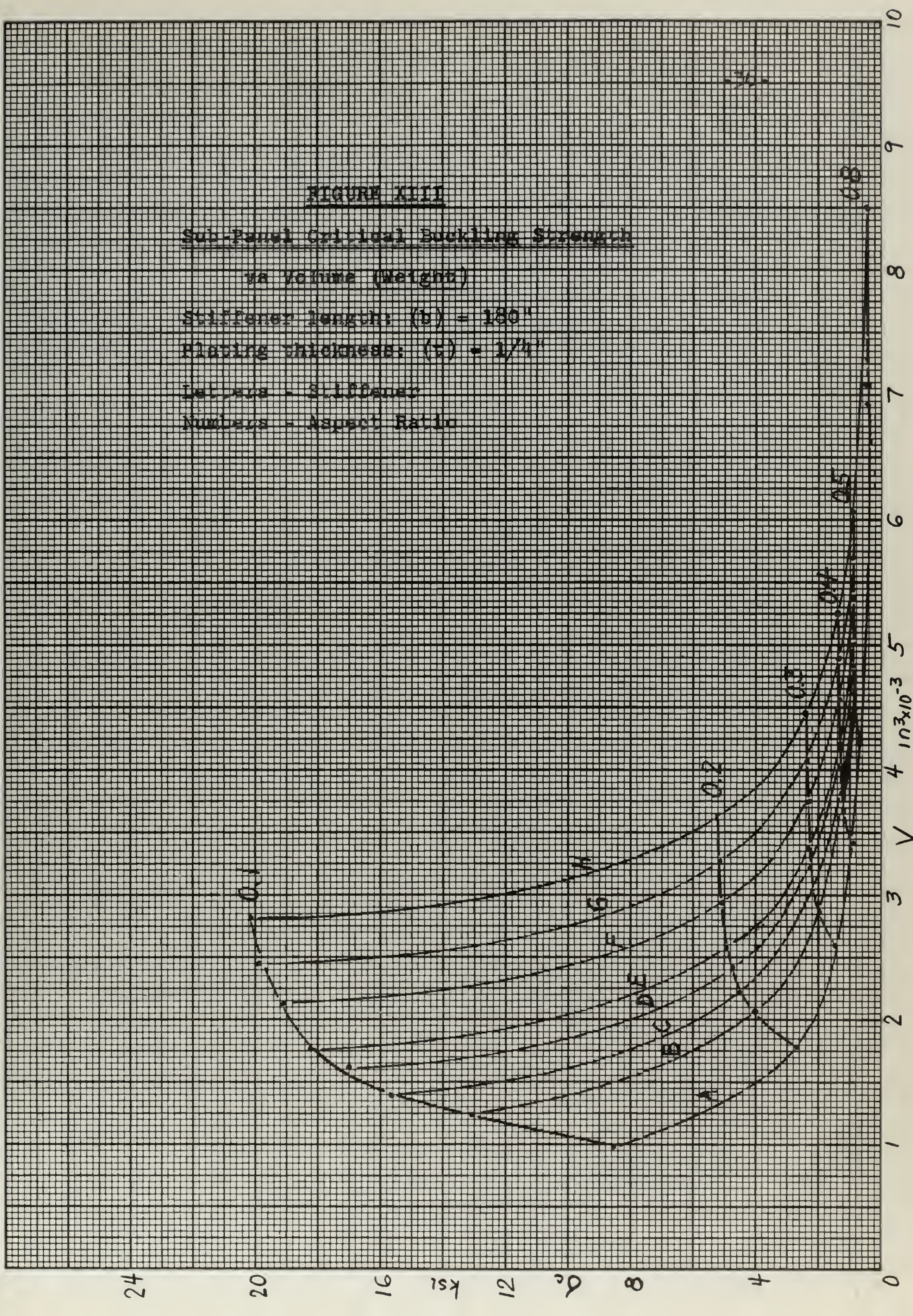


FIGURE XIII

Sub-Panel Critical Buckling Strength

vs Volume (Weight)

Stiffener length: (b) = 180"

Plating thickness: (t) = 1/4"

Letters - Stiffener

Numbers - Aspect Ratio

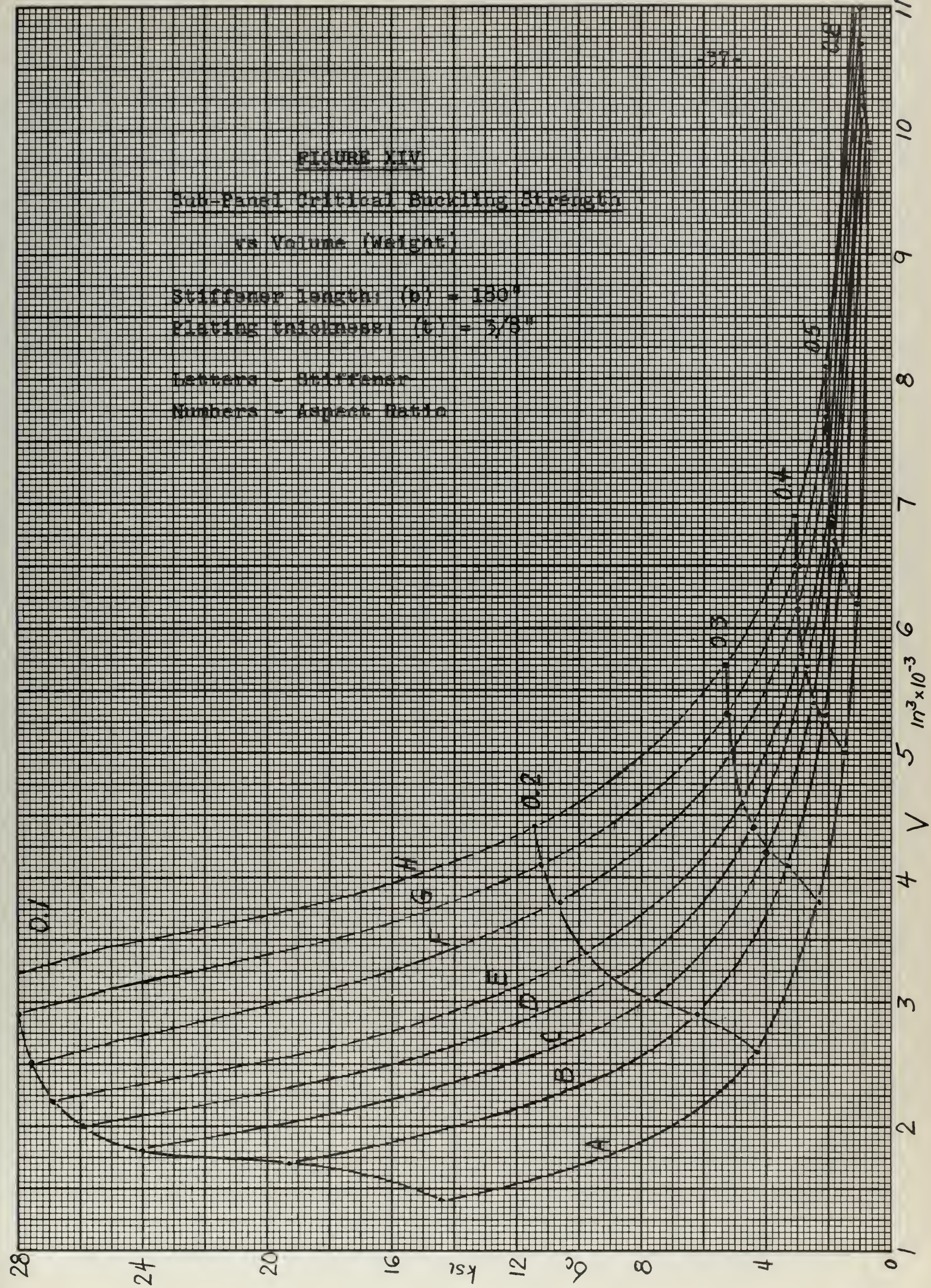


FIGURE XIV

Sub-Panel Critical Buckling Strength vs Volume (Weight)

Stiffener length: (b) = 180"

Plating thickness: (t) = 3/8"

Letters - Stiffener

Numbers - Aspect Ratio

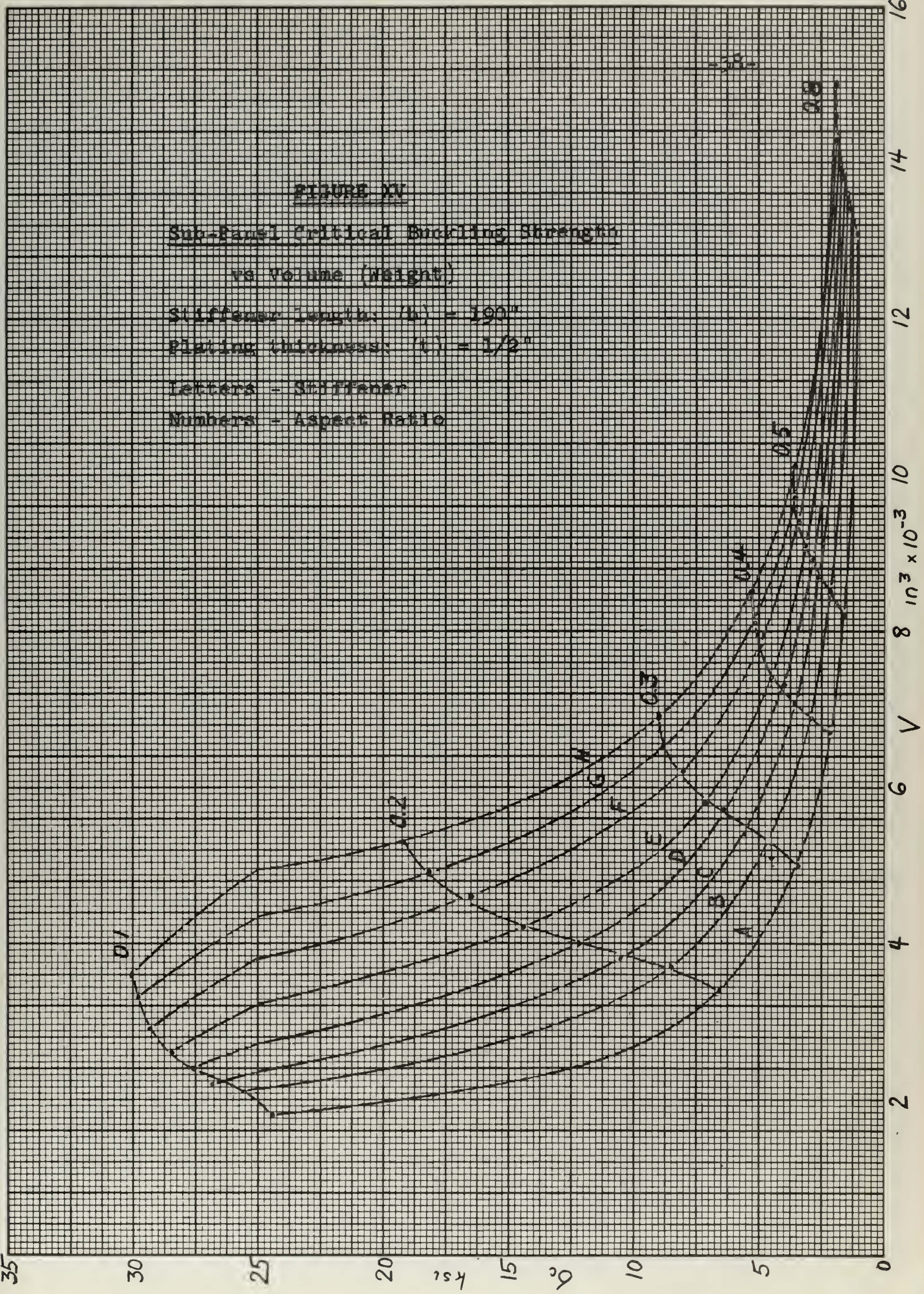


FIGURE XV

Sub-Panel Critical Buckling Strength

vs Volume (Weight)

Stiffener Length: (b) = 190"

Plating thickness: (t) = 1/2"

Letters - Stiffener

Numbers - Aspect Ratio

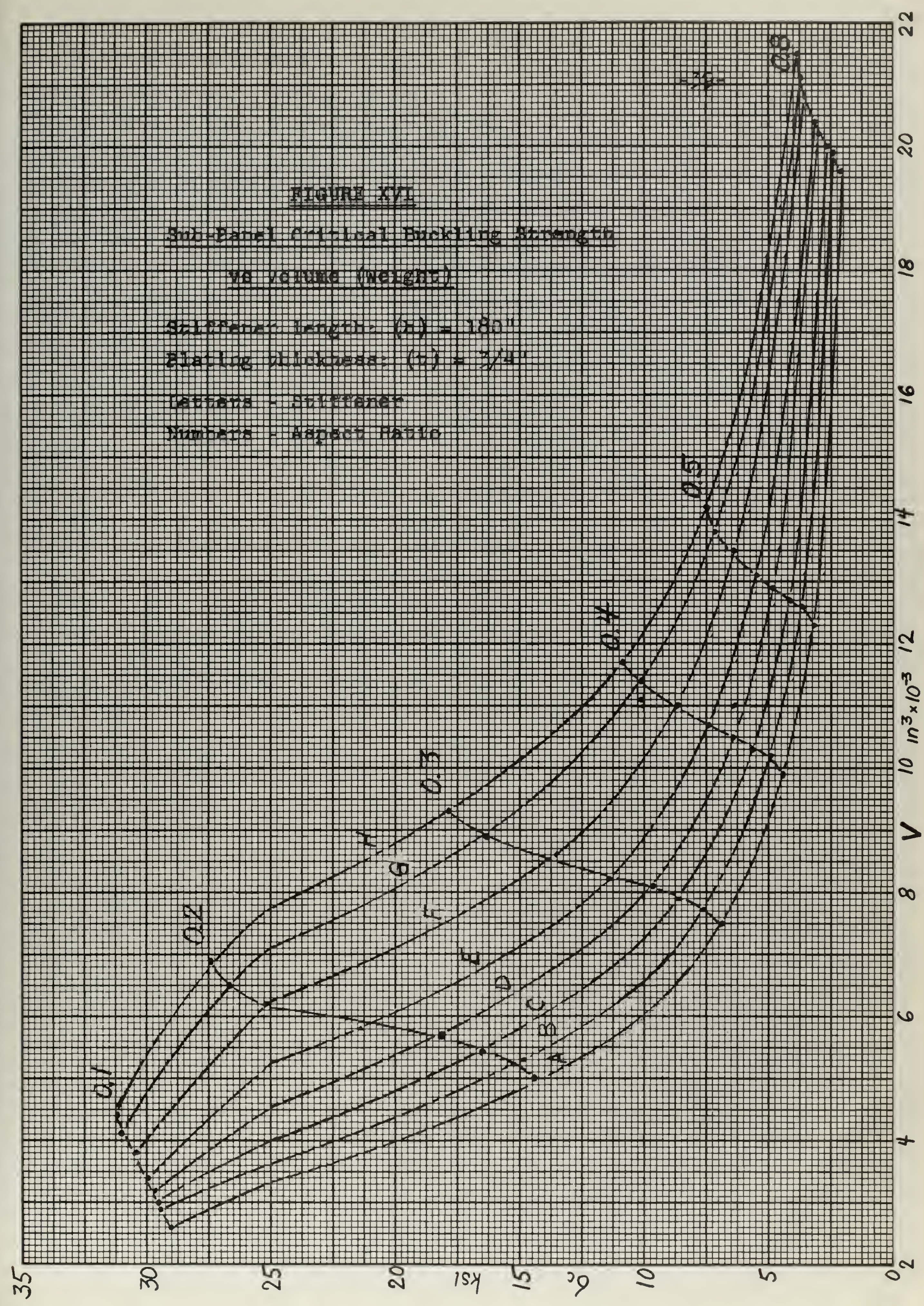


FIGURE XVI
Sub-Panel Critical Buckling Strength
vs Volume (Weight)
Stiffener Length: (a) = 180"
Plating Thickness: (t) = 3/4"
Letters - Stiffener
Numbers - Aspect Ratio
0.1
0.2
0.3
0.4
0.5
0.8
A
B
C
D
E
F
G
H
V in³×10⁻³
σc ksi
0
5
10
15
20
25
30
35
2
4
6
8
10
12
14
16
18
20
22

FIGURE XVII

Sub Panel Critical Buckling Strength

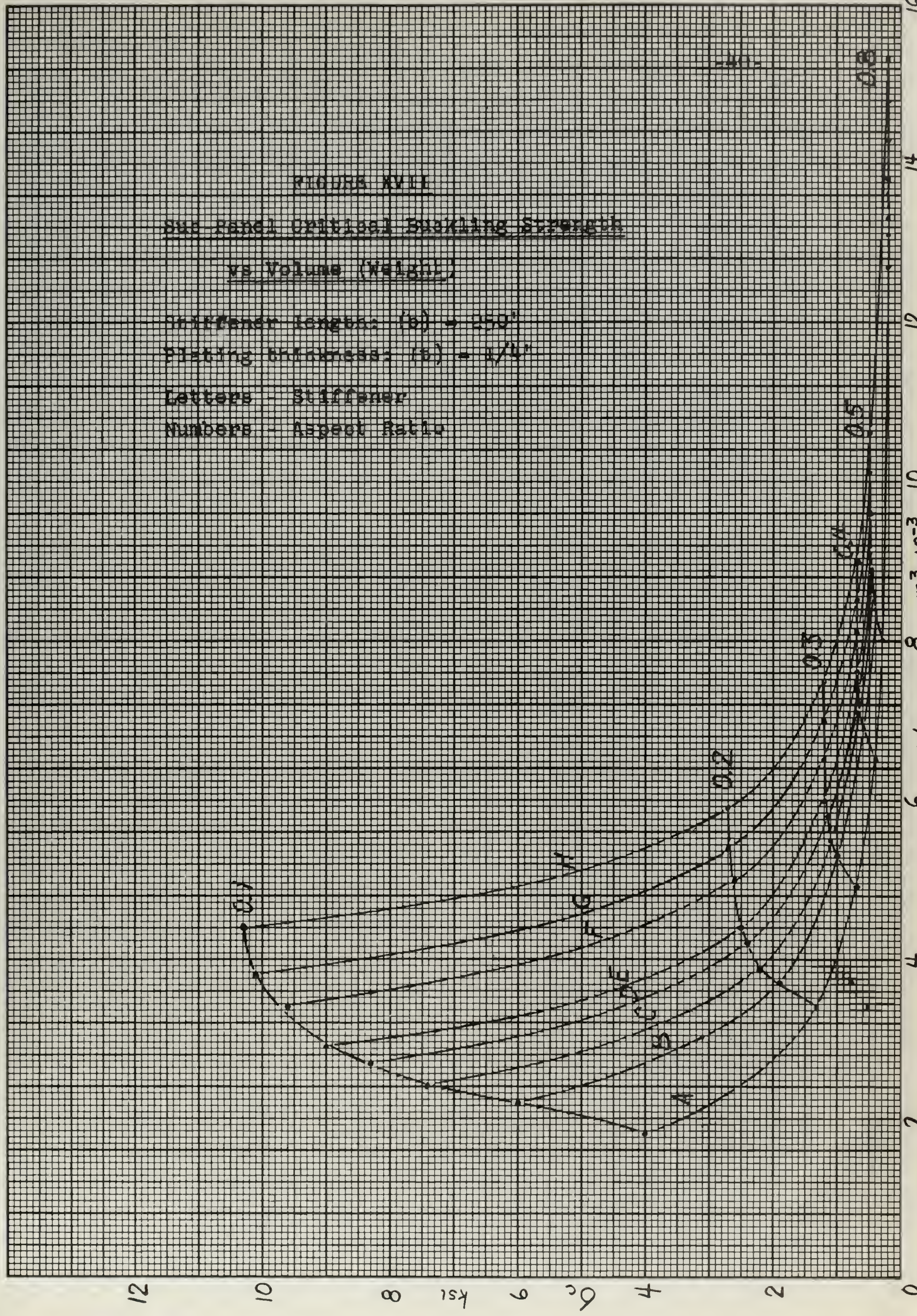
vs Volume (Weight)

Stiffener length: (b) = 250"

Plating thickness: (t) = 1/4"

Letters - Stiffener

Numbers - Aspect Ratio



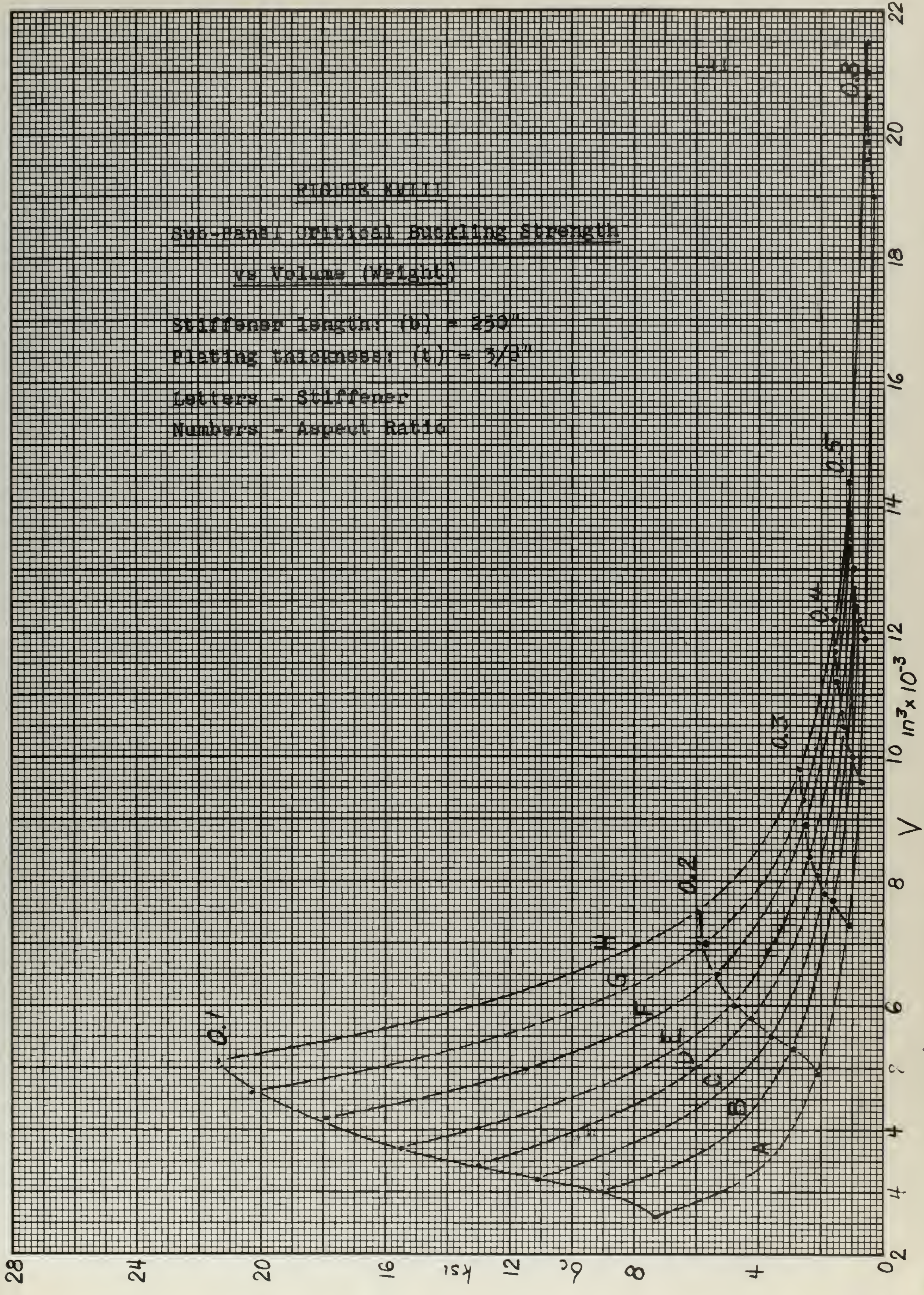


FIGURE XVIII
Sub-Panel Critical Buckling Strength
vs Volume (Weight)
Stiffener length: (b) = 250"
Plating thickness: (t) = 3/8"
Letters - Stiffener
Numbers - Aspect Ratio
0.1
0.2
0.3
0.4
0.5
0.8
A
B
C
D
E
F
G
H
V in³ x 10⁻³
ksi
0
4
8
12
16
20
24
28
2
6
10
14
18
22

TABLE XIX

Sub-Panel Critical Buckling Strength

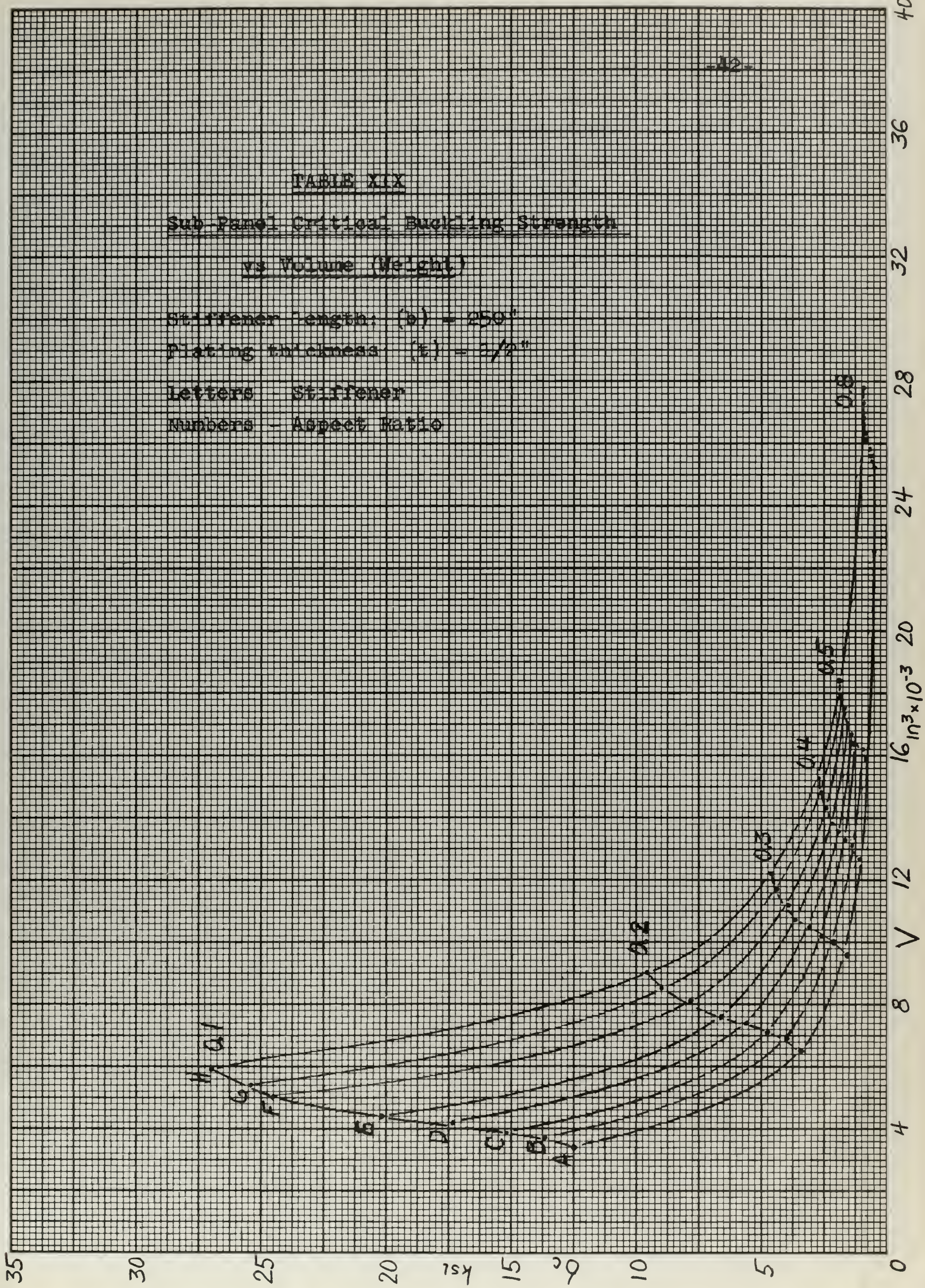
vs Volume (Weight)

Stiffener Length: (b) = 250"

Plating thickness: (t) = 1/2"

Letters - Stiffener

Numbers - Aspect Ratio



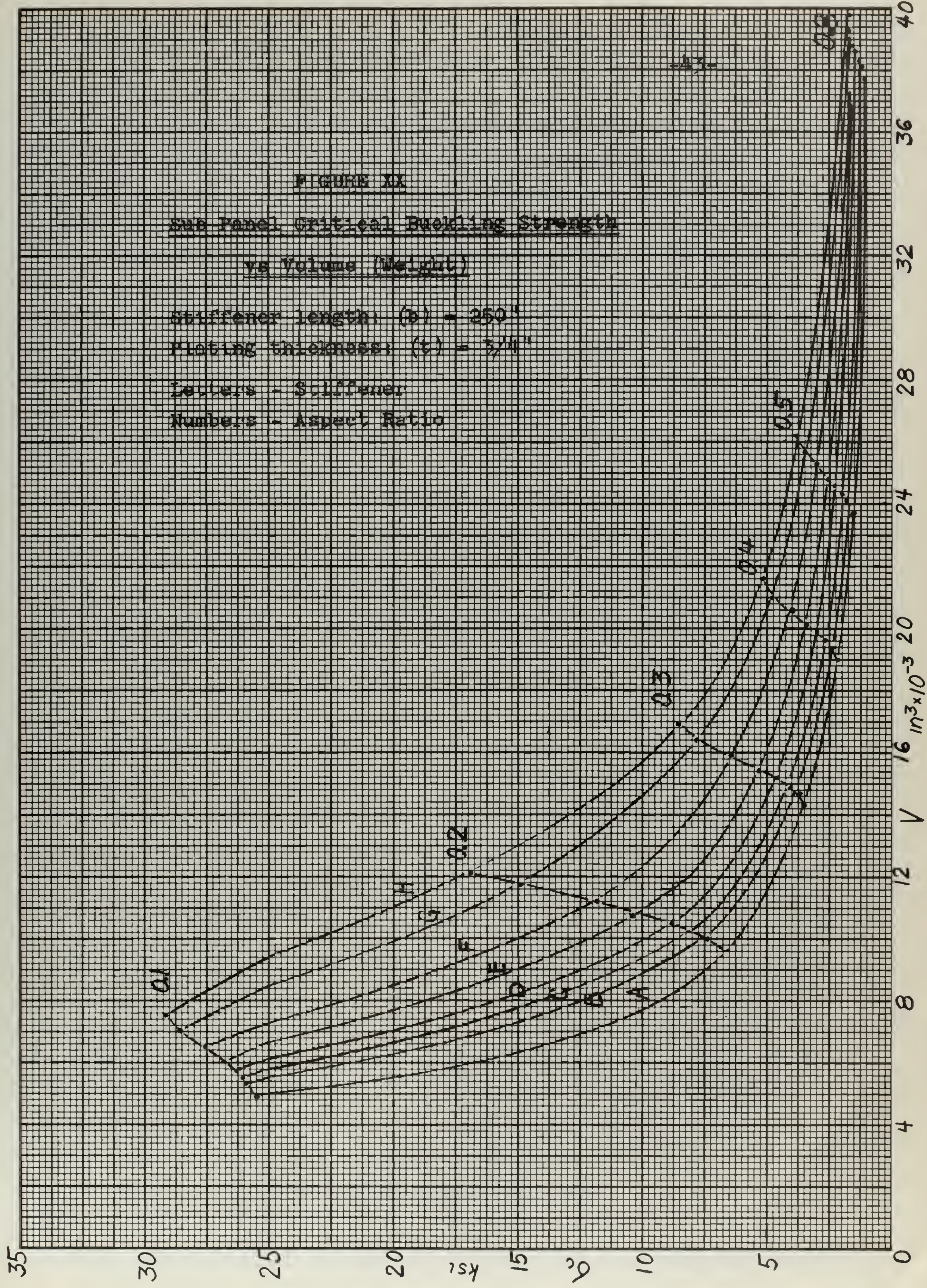


FIGURE XX
Sub Panel Critical Buckling Strength
vs Volume (Weight)
Stiffener length: (b) = 250"
Plating thickness: (t) = 3/4"
Letters - Stiffener
Numbers - Aspect Ratio
A
B
C
D
E
F
G
H
0.1
0.2
0.3
0.4
0.5
0.6
V $in^3 x 10^{-3}$
σ ksi
0
4
8
12
16
20
24
28
32
36
40
5
10
15
20
25
30
35

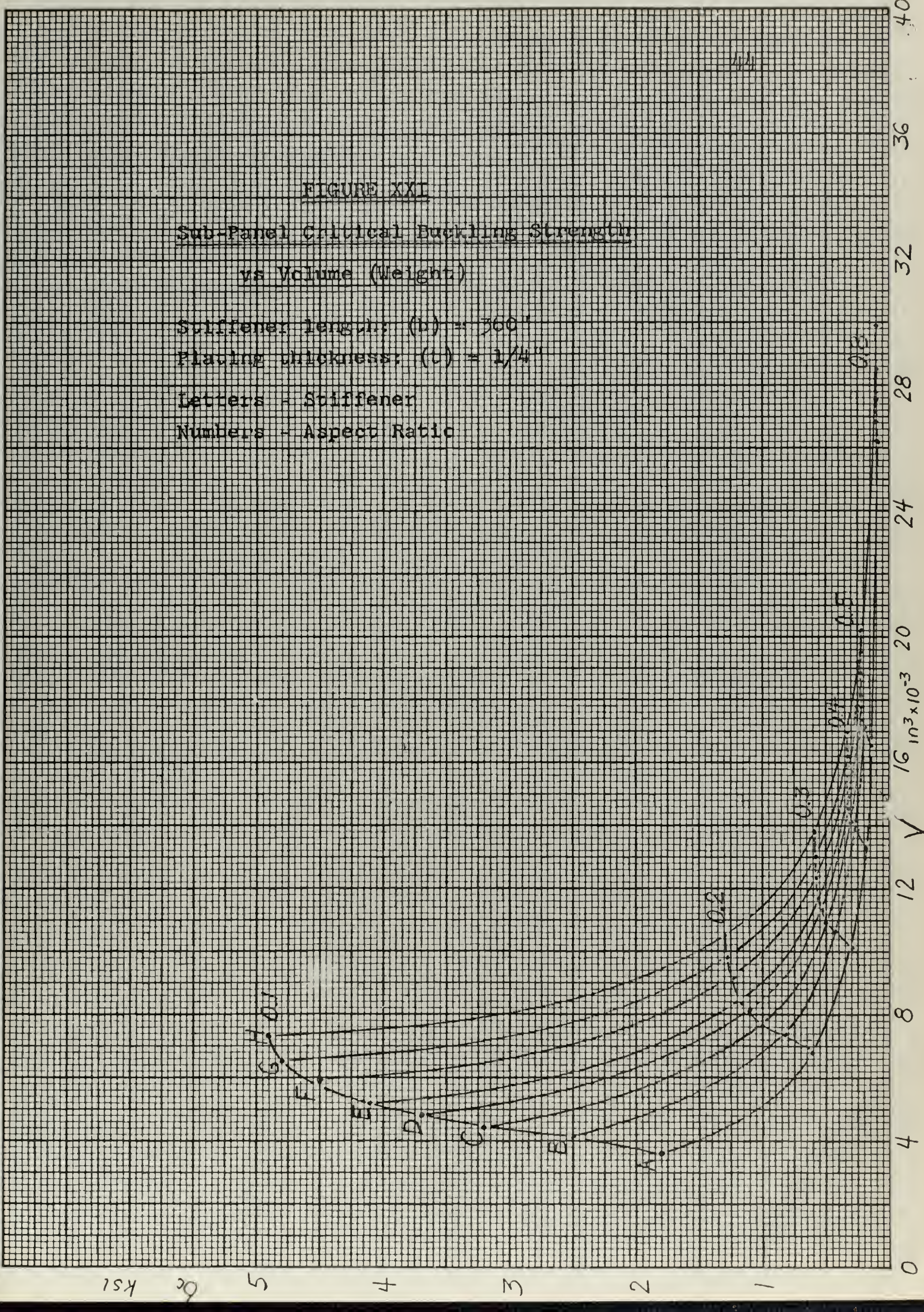


FIGURE XXI

Sub-Panel Critical Buckling Strength vs Volume (Weight)

Stiffener length: (b) = 360"

Plating thickness: (t) = 1/4"

Letters - Stiffener

Numbers - Aspect Ratio

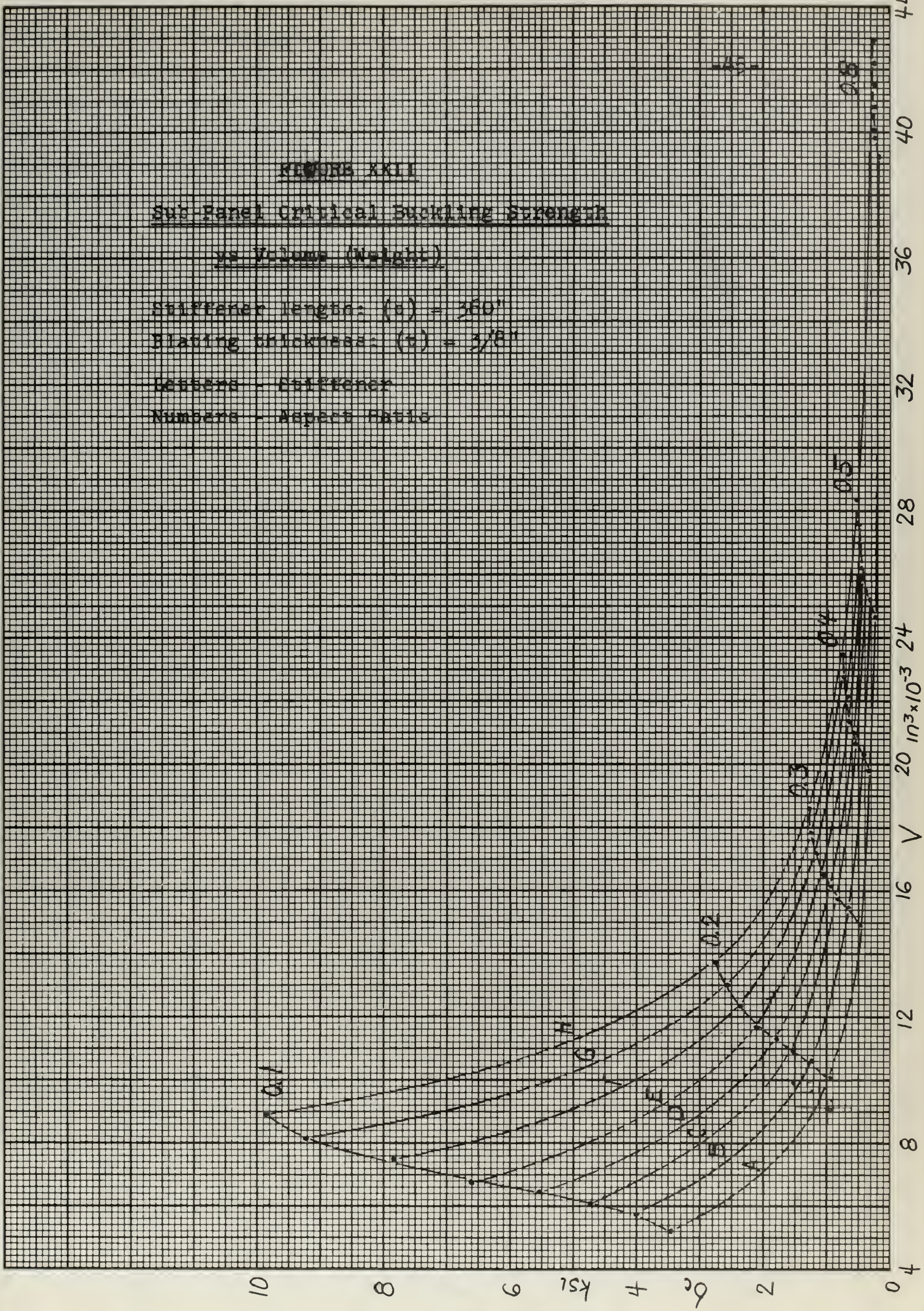


FIGURE XXII

Sub-Panel Critical Buckling Strength

vs Volume (Weight)

Stiffener length: (a) = 360"

Plating thickness: (t) = 3/8"

Letters - Stiffener

Numbers - Aspect Ratio

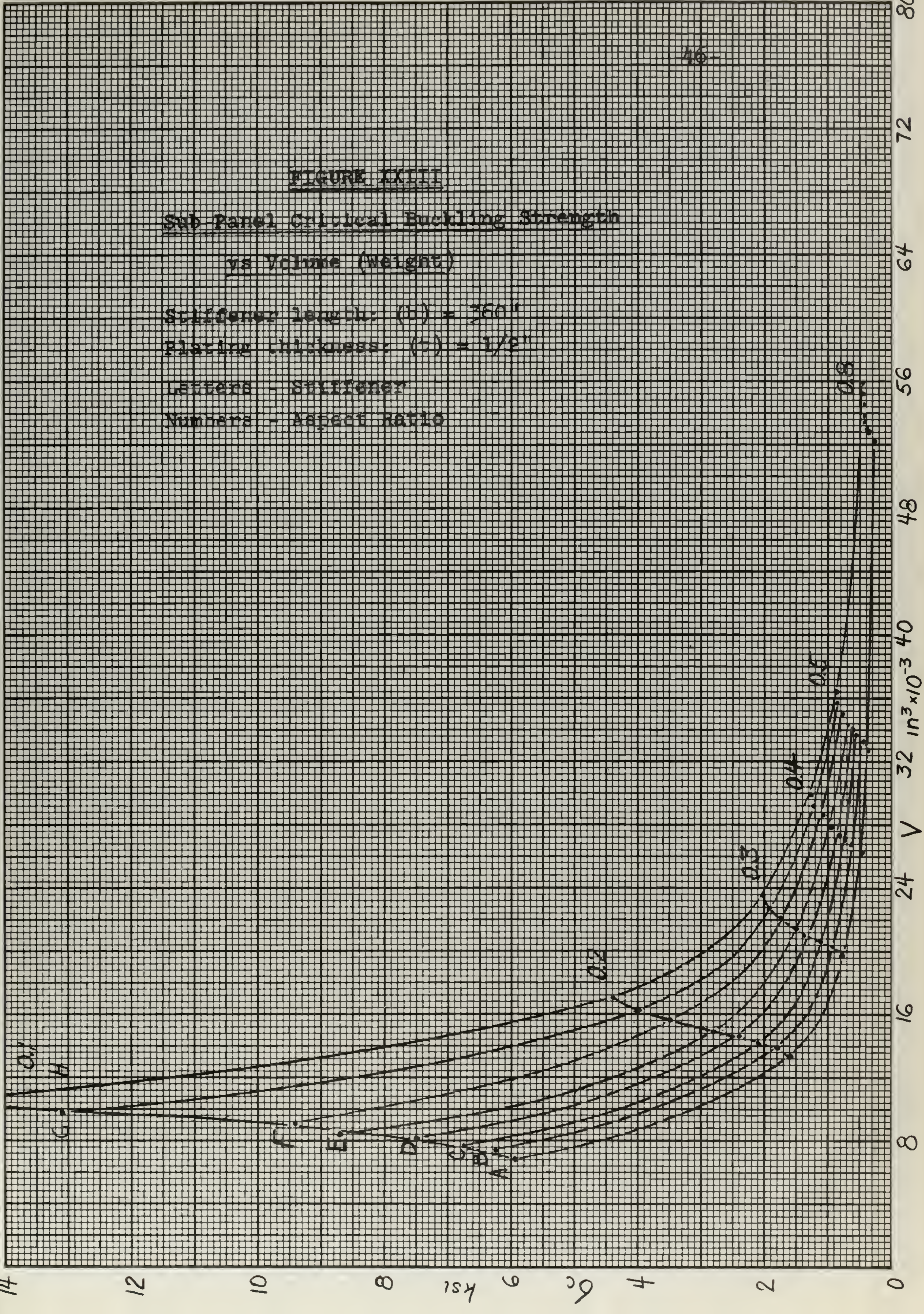
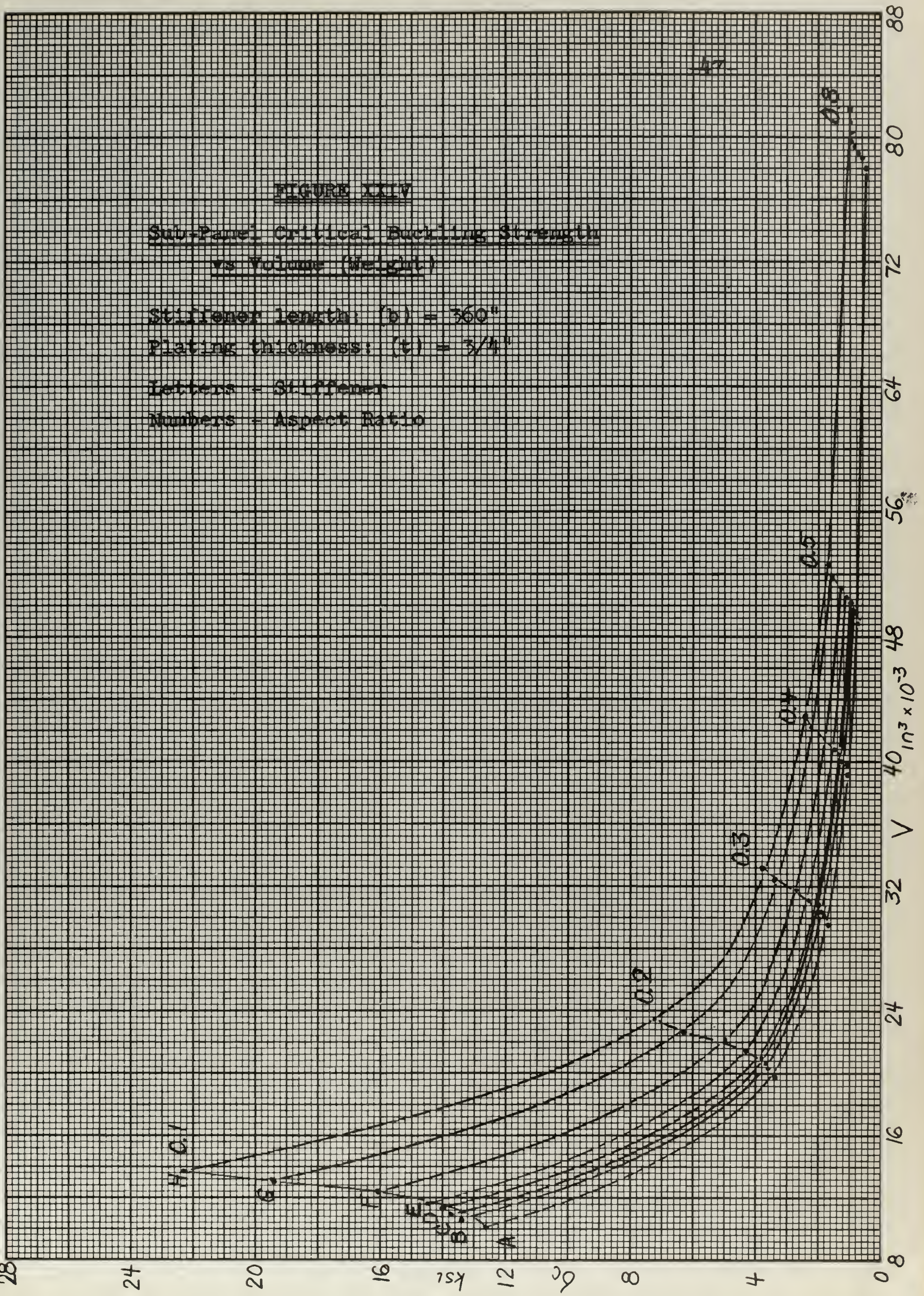


FIGURE XXIII
Sub Panel Critical Buckling Strength
vs Volume (Weight)
Stiffener length: (b) = 360"
Plating thickness: (t) = 1/2"
Letters - Stiffener
Numbers - Aspect Ratio
0.1
0.2
0.3
0.4
0.5
0.6
A
B
C
D
E
F
G
H
0
8
16
24
32
40
48
56
64
72
80
V in³×10⁻³
0
2
4
6
8
10
12
14
σc ksi


47
FIGURE XXIV
Sub-Panel Critical Buckling Strength
vs Volume (Weight)
Stiffener length: (b) = 360"
Plating thickness: (t) = 3/4"
Letters - Stiffener
Numbers - Aspect Ratio
0.8
0.5
0.4
0.3
0.2
H, 0.1
G
F
E
D
C
B
A
V $in^3 \times 10^{-3}$
σ_c ksi
0
8
16
24
32
40
48
56
64
72
80
88
0
4
8
12
16
20
24
28

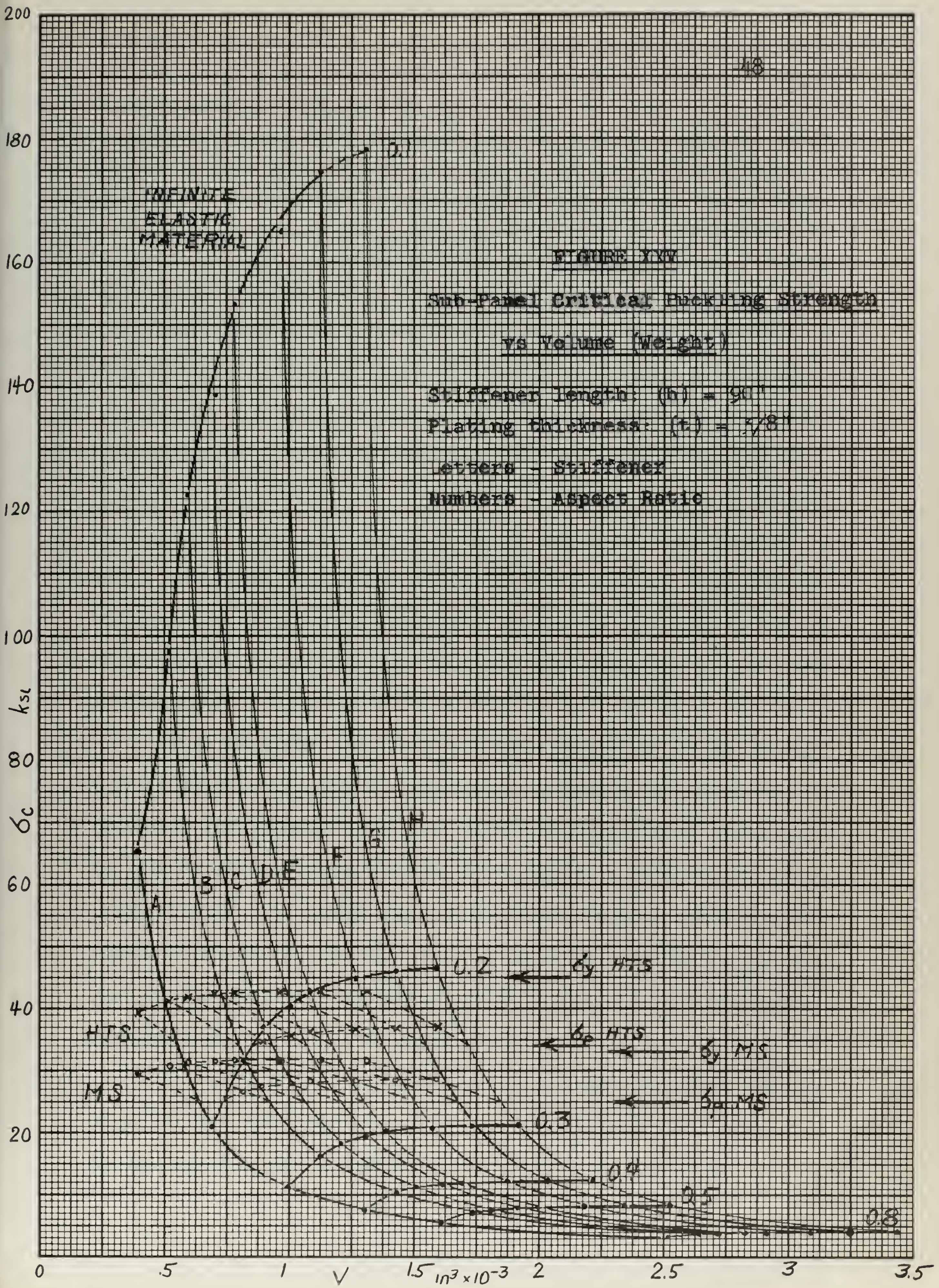


FIGURE XXV
Sub-Panel Critical Buckling Strength
vs Volume (Weight)
Stiffener Length: (h) = 90"
Plating Thickness: (t) = 5/8"
Letters - Stiffener
Numbers - Aspect Ratio
INFINITE ELASTIC MATERIAL
0.1
0.2
0.3
0.4
0.5
0.8
A
B
C
D
E
F
G
H
HTS
MS
σ_y HTS
σ_p HTS
σ_y MS
σ_p MS
σ_c ksi
V in^3 × 10^{-3}
200
180
160
140
120
100
80
60
40
20
0
.5
1
1.5
2
2.5
3
3.5

DISCUSSION

This thesis is intended as a small contribution to the development of an overall method of design synthesis. It studies but one condition of loading. The results are presented in a fashion suitable for use with a suggested design method. It does not cover the full range of values of plating thickness and stiffener scantlings in common use. A fundamental difficulty lies in meeting the requirements of the assumption that the amount of loading is known. Such is not presently the general case for ship gross panel structures.

This study is one more building block to which others can be added in order to arrive at a family of analyses covering all conditions. It is to be hoped that correlation of this analysis with others can include a standardization of approach. It would appear that development of a single overall solution for the general case of loading is at least far in the future, certainly quite difficult and perhaps impossible by presently known techniques.

If our problem had been one of simple buckling the design curves obtained would follow the general shape of the so-called Tangent Modulus curves (plots of critical stress

vs aspect ratio in Ref. (3), page 350). However our problem is complicated by the restraint provided by the stiffeners. Furthermore, the selection of stiffeners is not analytic but from available commercial sections. These two factors materially alter the character of the curves.

The results confirm expectations. In the theoretical case of very small sub-panel aspect ratio the critical buckling stress approaches the yield stress of the material while at large sub-panel aspect ratios it approaches zero.

Cursory consideration would lead to the conclusion that a combination of heavy stiffener with thin plate would be a good solution. The heavy stiffener would increase the restraint, therefore increase the value of the critical buckling stress. The thin plate would mean a lighter structure. However, our results confirm the tenet that good structural designs possess balance.

The concept of "proper balance" suffers because there is no general formulation or quantitative description of it. Our analysis gives indication of where proper balance exists for one type of structure under one condition of loading. It confirms the thumb rule that stiffener weight should about equal plating weight.

Given a gross panel of certain dimensions, from a minimum weight viewpoint, it would be desirable to have

a minimum number of sub-panels (hence stiffeners) since this would imply smaller total weight of stiffeners (if stiffeners were of constant size). However, from a strength viewpoint small aspect ratio hence large number of sub-panels is desirable to increase buckling strength of the panel.

Similarly, an increase in plate thickness would increase the critical buckling stress, but an undue thickness results in rapid weight increase.

Stiffener scantlings have a major influence on the buckling strength of the structure. The results indicate that there is a certain limit beyond which an increase in stiffener scantling is no longer economical. Many of the stiffener family curves (constant aspect ratio in any one of the design graphs, Figures I through XXIV) show a distinct reduction in slope of buckling stress versus sub-panel weight within the range studied. It would appear that an optimum strength-weight ratio occurs near this knee.

The relationship between buckling stress and design stress of the structure and the proportional limit and yield stress of the material utilized is a matter of discretion with the designer. Numerous thumb rules have been proposed. It would seem logical that the buckling stress should be at least equal to the design stress, and that the buckling and/or design stress should be related to the yield stress by the desired safety factor. No assumption as to safety factor is included in our analysis.

The material selected for the application of our analysis (medium steel) has a rather low proportional limit and yield stress (25 ksi and 33 ksi, respectively). Its modulus of elasticity is 29,600,000 psi. Its properties above the proportional limit are given in Table III. The selection was made arbitrarily on a basis of its more prevalent total utilization. Buckling stress values for stronger materials such as high tensile, special treatment and high yield steels could be arrived at very rapidly.

Note that the curves of buckling stress versus subpanel weight for constant stiffener size are severely distorted above the proportional limit. (See Fig. XXV). This applies to a limited degree to some of the constant aspect ratio curves as well, where they cross the 25 ksi line. However, this by no means weakens the argument previously mentioned concerning an optimum balance, since similar knees occur in constant aspect ratio curves well removed from the proportional limit.

Direct comparison of results between this study and that of Harlander could be misleading since different conditions of loading are considered. We will use the example which Harlander presented (Ref. 8, page 61). In order to have a basis of comparison, the safety factor of 2.5 must be applied to the design stress to arrive at the buckling stress, contrary to our proposed use of the factor.

[illegible]

Harlander studied a gross panel with overall dimensions of b = 10 ft. and L = 10 ft. A total design load on edge of 1000 kips was applied. A safety factor of 2.5 was assumed. The optimum results obtained by his iterative method are:

Aspect Ratio of Sub-panel	Plate	Stiffener (Angle)	Total Weight
0.1	5/16 in	7x4x7/16	2500 lbs.

Using the same gross panel dimensions, loading and factor of safety but with stiffeners parallel to the loaded edges instead of perpendicular, we obtain the following solutions by our proposed iterative design method:

Aspect Ratio	Plate	Stiffener	Weight
0.1	3/4 in	$4x2\frac{1}{4}x3\frac{1}{4}$	3360 lbs
0.2	3/4 in	6x4x11	3630 lbs
0.3	3/4 in	12x9x38	3980 lbs

It is assumed that the simplifying assumptions and approximations in our two methods affect the solutions equally, and that differing stiffeners have no effect. It is apparent that transversely framed panels will be somewhat heavier than longitudinally framed panels, both loaded with edge compression only. The effect of additional types of loading and of other than idealized boundary conditions is problematical. The desirability

of integrating the structure of the gross panel under consideration with adjacent structure will certainly also affect the choice of stiffener orientation.

CONCLUSIONS

It is concluded that:

1. Aspect ratio of sub-panels should be small. Inspection of the design curves indicates that for normal proportions the aspect ratio should not exceed 0.3.

2. Short stiffener span is preferable to long span. We have not considered the weight of structure necessary to divide the gross panel into smaller gross panels however.

3. The optimum design (proper balance) will have stiffener weight about equal to plate weight.

4. The buckling strength of light-weight high-strength structures increases proportionally to the yield stress where the buckling stress is above the proportional limit of the lower-strength material. It had heretofore been feared that the advantages of high-strength materials could not be obtained due to limited buckling strength.

5. Transversely stiffened gross panels will apparently exceed longitudinally stiffened panels in weight.

RECOMMENDATIONS

It is recommended that:

1. A continuing study of the gross panel analysis of ship stiffener-plating structures be pursued.

2. A single integrated solution for the general case be aimed for.

3. Interim "building block" methods applicable to only one condition of loading or panel proportion be standardized as to method of design utilization.

4. If practicable, the results of such interim methods be non-dimensionalized for comparative general application rather than the specific dimensional.

5. Subject to evaluation in actual use, this analysis should be extended to cover the full ranges of plate thickness and Tee stiffener scantlings and to different types of stiffeners. Such extension should include more closely spaced data intervals in the areas of interest outlined (aspect ratios 0.05 to 0.20, plate thickness 3/16" to $1\frac{1}{2}$").

6. Quantitative analysis of the comparative weight requirements for longitudinally vs transversely stiffened

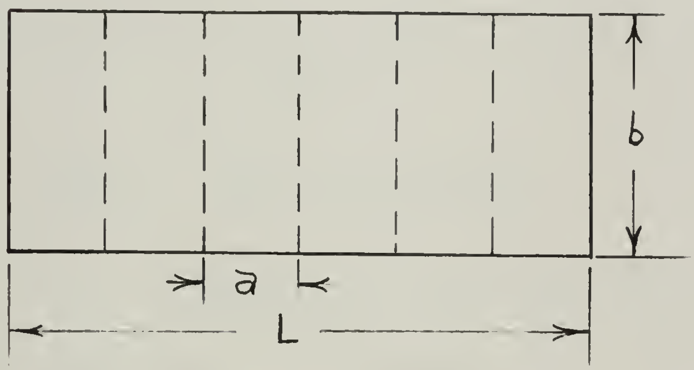
gross panels be made. This will require:

a. Similar stiffeners in each case.

b. Closer spacing of data intervals as mentioned above.

APPENDIX

Derivation of Working Formulae for Rectangular Plates in Longitudinal Compression, Elastically Restrained at the Loaded Edges, Simply Supported at the Unloaded Edges.



E = modulus of elasticity of steel (psi).

t = thickness of plating, in.

a = stiffener spacing, in.

$\bar{k}$ = plate factor

ν = Poisson's Ratio

ζ = coefficient of restraint

b = length of stiffener, in.

I_{yy} = moment of inertia of stiffener about its y-y axis, in^4

d = depth of stiffener, in.

K = torsion constant of stiffener, in^4

For a single sub-panel, the critical stress σ_c is given by:

$$\frac{\sigma_c}{\tau} = \frac{\pi^2 E}{12(1-\nu^2)} \left(\frac{t}{a}\right)^2 \bar{k}$$

But

$$\pi = 3.1416$$

$$E = 29{,}600 \text{ ksi}$$

$$\nu = 0.3$$

and

$$\frac{t}{a} = c$$

Then, $$\frac{\sigma_c}{t} = 26{,}750\ c^2 \bar{k}$$

$\bar{k}$ is a function of the aspect ratio $\alpha = a/b$ and of the coefficient of restraint, ζ , and these values are shown in Table I.

The coefficient of restraint is given by the following

$$\zeta = \frac{t^3\ b^2}{53.9\left(\frac{\pi^2 I_{yy} d^2}{b^2} + \frac{K}{2.6}\right)}$$

and by the use of the auxiliary functions $c = \frac{t}{a}$, $\alpha = \frac{a}{b}$ we obtain

$$\zeta = \frac{a^4\ c^3}{53.9\ \alpha^2\left(\pi^2 \frac{I_{yy} d^2 \alpha^2}{a^2} + \frac{K}{2.6}\right)}$$

$$\zeta = \frac{a^4\ c^3}{53.9\,\alpha^2 (R_1 + R_2)}$$

Where $R_1 = \pi^2 \frac{I_{yy} d^2 \alpha^2}{a^2}$; $R_2 = \frac{K}{2.6}$

The values of K are given in Table II.

Once the value of $\frac{\sigma_c}{t}$ has been obtained, by means of Table III we obtain the critical buckling stress σ_c.

TABLE I

Plate Factor k for Short Plates in Compression, Elastically Restrained at the Loaded Edges

From Ref. (1), Table 40, pg. 436

Coefficient of Restraint ζ	Aspect Ratio α = a/b 0.1	0.2	0.3	0.4	0.5	0.6	0.8	1.0
0,clamped	4.02	4.08	4.19	4.34	4.55	4.82	5.59	6.74
0.05	3.35	3.40	3.51	3.62	3.88	4.15	4.94	6.18
0.10	2.88	2.98	3.04	3.18	3.42	3.70	4.51	5.78
0.15	2.56	2.62	2.72	2.86	3.10	3.38	4.21	5.48
0.20	2.32	2.38	2.49	2.62	2.86	3.15	3.98	5.26
0.25	2.15	2.21	2.32	2.45	2.68	2.97	3.80	5.09
0.30	2.01	2.07	2.18	2.32	2.55	2.84	3.67	4.98
0.35	1.90	1.96	2.08	2.22	2.44	2.73	3.57	4.89
0.40	1.81	1.88	1.99	2.13	2.35	2.65	3.48	4.80
0.45	1.74	1.81	1.91	2.06	2.28	2.58	3.41	4.73
0.50	1.69	1.75	1.85	2.00	2.23	2.52	3.35	4.68
1.00	1.39	1.46	1.55	1.71	1.92	2.22	3.06	4.37
2.00	1.22	1.27	1.38	1.55	1.76	2.04	2.89	4.17
∞, simply supported	1.02	1.08	1.19	1.35	1.56	1.85	2.69	4.00

TABLE II

Properties of Stiffeners

From Ref. (6)

	Stiffener	Wt./Ft. lbs.	Area of Section in^2	Depth of Tee (in)	I_{xx} (in^4)	I_{yy} (in^4)	Flange Width (in)	Average Thickness (in)
A	4"x2¼"x3.25	3.25	0.96	4.00	1.59	0.17	2.28	0.189
B	5"x4"x8.5	8.50	2.49	5.06	6.07	1.73	4.01	0.329
C	6"x4"x11	11.00	3.24	6.16	11.7	2.27	4.03	0.424
D	7"x6-3/4"x15	15.00	4.41	6.93	19.0	8.77	6.73	0.383
E	8"x7"x18	18.00	5.30	7.93	30.7	11.10	6.99	0.428
F	9"x7½"x25	25.00	7.35	9.00	53.9	18.60	7.50	0.570
G	10"x8¼"x31	31.00	9.12	10.49	93.7	26.60	8.24	0.615
H	12"x9"x38	38.0	11.18	11.95	151.7	38.3	8.99	0.682

(continued)

	Stiffener	K (in^4) Torsion Coeff.	Stem T_W Thickness (in)	Depth of Stem D (in)	Depth of Stem / (Stem Thickness)	Width of Flange / (Stem Thickness)
A	4"x2¼"x3.25	2.53	0.135	3.81	28.2	16.9
B	5"x4"x8.5	8.51	0.24	4.73	19.7	16.7
C	6"x4"x11	23.70	0.26	5.74	22.1	15.4
D	7"x6-3/4"x15	25.47	0.27	6.55	24.3	24.9
E	8"x7"x18	26.50	0.36	7.50	25.0	23.4
F	9"x7½"x25	72.05	0.36	8.43	23.4	20.8
G	10"x8¼"x31	139.10	0.40	9.87	24.6	20.6
H	12"x9"x38	211.20	0.44	11.27	25.7	20.5

TABLE III

Determination of the Critical Stress σ_c for Medium Steel, Kips/in^2 (σ_p = 25 Kips/in^2, σ_y = 33 Kips/in^2)

From Ref. (1), Table 41, pg. 438

σ_c/τ	σ_c	σ_c/τ	σ_c
25	25.0	40.0	28.00
25.5	25.17	45.0	28.54
26.0	25.32	50.0	29.00
26.5	25.46	55.0	29.36
27.0	25.60	60.0	29.67
27.5	25.73	70.0	30.14
28.0	25.86	80.0	30.50
28.5	25.98	90.0	30.78
29.0	26.10	100.0	31.00
29.5	26.22	120.0	31.33
30.0	26.33	140.0	31.57
31.0	26.54	160.0	31.74
32.0	26.74	180.0	31.88
33.00	26.94	200.0	32.00
34.0	27.12	250.0	32.20
35.0	27.29	300.0	32.32
36.0	27.45	400.0	32.48
37.0	27.60	500.0	32.60
38.0	27.74		
39.0	27.87		

Limiting Criteria in the Selection of Stiffeners

Several limiting criteria exist (7). The limitations of the web depth to thickness ratio and the flange width to thickness ratio are in accordance with structural practice. These criteria are based principally on column load only (i.e., purely axial compression) which is not the case in our problem. To follow them is in line with structural good practice, however, The first limiting criteria is that the depth to thickness ratio of the web should not be greater than 60. The second is that the ratio of flange width to web thickness should not exceed 30 nor should it be less than 10. The high limit of 30 comes from the fact that an excessively wide flange would tend to buckle at the edges before yield point stress in the flange is developed. The low limit is due to the fact that a narrow flange will not develop full buckling strength of the web.

The characteristics of the selected stiffeners are shown in Table II. It can be seen that they meet these limiting criteria.

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

TABLE IV

Typical Calculation

$\alpha = a/b$	0.1							
$c = t/a$	2.085×10^{-2}							
t	1/4"							
a	12"							
b	120"							
Section	A 4"x2¼"x3¼#	B 5"x4"x8½#	C 6"x4"x11#	D 7"x6¾x15#	E 8"x7"x18#	F 9"x7½"x25#	G 10"x8¼"x31#	H 12"x9"x38#
c^3	9.05×10^{-6}							
R_2	0.97	3.28	6.32	9.8	16.2	27.7	53.5	81.3
$I_{yy}\,d^2$	2.72	44.3	86.4	421	698	1508	2925	5480
α^2	10^{-2}							
a^2	144							
$\frac{\pi^2\alpha^2}{a^2}$	6.85×10^{-4}							
R_1	.0019	.03	.0591	.288	.478	1.034	2.00	3.76
R_1+R_2	0.97	3.31	6.38	10.1	16.7	28.7	55.5	85.1
① $\frac{c^3}{539\alpha^2}$	1.68×10^{-5}							
② a^4	2.075×10^{4}							
λ = ①×②	0.348							
$\rho = \frac{\lambda}{R_1+R_2}$	0.359	0.1052	0.0546	0.0345	0.0209	0.0121	0.0063	.0041
$\bar{k}$	1.89	2.85	3.27	3.49	3.68	3.80	3.90	3.93
c^2	4.34×10^{-4}							
σ_c/τ	21.9	33.05	38.0	41.6	42.7	44.1	45.2	45.6
$\sigma_{c\ M.S.}$	21.9	26.95	27.7	28.2	28.3	28.4	28.5	28.6
abt	360							
Ab	113	299	389	529	636	882	1096	1340
V	473	659	749	889	996	1242	1456	1700

DESIGN METHOD

1. Accept a predetermined series of stiffeners.
2. Know gross panel dimensions.
3. Know loading in plane of plate, perpendicular to stiffener orientation, and total amount. Hence can infer dimension across loading direction - b.
4. Arbitrarily select n, number of stiffeners, hence determine a and a/b.
5. Arbibrarily select t, plating thickness. From b and t calculate σ_x from known loading.
6. Enter graph. Ensure that σ_c for selected a/b and t is in excess of σ_x. Note weight of stiffener and plate.
7. Compute weights of gross panel for each of the several values of n and t. Select least weight solution.

ILLUSTRATIVE EXAMPLE

Given: Gross panel 10" square, loaded on two opposite edges with uniform compression in plane of plate of 750,000 pounds. Stiffeners of this thesis to be arranged transverse to load.

Solution:

1. Acceptable.

2. Panel 10' x 10'

3. b = 120" L = 120"

4. Assume values of n = 2, 3, 5, 10

Assume values of n =	2	3	5	10
Therefore $a = \frac{L}{n}$ =	60	40	24	12
And $\alpha = \frac{a}{b}$ =	.5	.333	.2	.1

5. Select plating thickness, $t_{(in)}$ = 1/4, 3/8, 1/2, 3/4

Then $\sigma_x \, (Ksi) = \frac{P}{bt} = \frac{750}{120t}$ = 25, 16-2/3, $12\frac{1}{2}$, 8-1/3

6.

$\alpha \backslash t$	1/4"	3/8"	1/2"	3/4"
.5	--	--	--	B-5.80
.333	--	--	D-2.95	A-3.60
.2	--	C-1.44	A-1.55	A-2.27
.1	B-.66	A-.63	A-.83	A-1.20

Letter is stiffener.

Number is volume.

Blanks mean no σ_c in excess of σ_x.

7. For each α select the least-volume solution from above. Convert these to weight of gross panel. $W = .284\ n\ V$

α	t	V	Stiffener	Panel Weight
.1	3/8	.63	A	1,790 lb.
.2	3/8	1.44	C	2,040
.333	1/2	2.95	D	2,510
.5	3/4	5.8	B	3,300

Hence 4" x $2\frac{1}{4}$" x $3\frac{1}{4}$ lb. Tee stiffeners will be spaced 12" apart on 3/8" plating for a gross panel weight of 1,790 pounds.

BIBLIOGRAPHY

(1) Timoshenko, S. "Theory of Elastic Stability". McGraw-Hill Book Co.,Inc., New York (1936).

(2) Timoshenko, S. "Theory of Plates and Shells" McGraw-Hill Book Co., Inc., New York (1940).

(3) Bleich, F. "Buckling Strength of Metal Structures" McGraw-Hill Book Co., Inc., New York (1952).

(4) Ros, M. and A. Eichinger. "Final Report of the First Congress of the International Association of Bridge Structural Engineers" Paris, (1932).

(5) Timoshenko, S. "Theory of Elasticity" McGraw-Hill Book Co.,Inc., New York (1934).

(6) "Steel Construction". American Institute of Steel Construction, New York, (1955).

(7) "Design Data for Tee Stiffeners". Navy Department, Bureau of Ships, No. 017969

(8) Harlander, Leslie A. "Optimum Plate Stiffener Arrangement for Various Types of Loading" Thesis, M.I.T., (1955).